THE

ELEMENTS OF ARITHMETIC;

FOR

SCHOOLS AND ACADEMIES,

IN WHICH

DECIMAL AND INTEGRAL ARITHMETIC ARE COMBINED,
AND TAUGHT INDUCTIVELY,

ON THE SYSTEM OF PESTALOZZI.

PART SECOND.

BY PLINY E. CHASE, A.M.

PHILADELPHIA:
URIAH HUNT AND SON,
[illegible] NORTH FOURTH ST.
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PREFACE.

The improvements which have been made within a few years, in the mode of teaching Arithmetic, and the farther improvements which may result from a philosophical arrangement, and a more satisfactory explanation of some principles of the science, were the inducements which led to the compilation of the present series. In the preparation of the original manuscript, I was guided by a knowledge of the wants of my own pupils; and after I had been advised to publish the result of my labours, I studied to acquaint myself with the best European and American treatises on the subject, and to derive from experienced practical teachers* such information as would assist the pupil, by removing the most formidable obstacles from his path.

Particular care has been taken to illustrate all the principles of Fractions, and to simplify the general arrangement, by treating in connection all subjects of a similar nature. The work has in this manner been greatly condensed, and an opportunity afforded for giving full explanations of every difficult point, and embracing within a small compass, a greater variety of subjects than is contained in any of our present treatises.

The most important ORIGINAL feature of the work, is the union of decimals with integers in the simple rules,—a mode of instruction that is rapidly gaining ground, and is sanctioned by the practice of many most successful teachers. Its *practicability* having been fully tested, the advantage derived from dispensing with many of the old rules, which are entirely superfluous, and tend to retard the progress of the pupil, will be readily appreciated.

The Rule of Three has lost much of its ancient reputation, and some of its applications have been very properly dismissed from our modern text-books. Still there are many examples which involve the theory of Proportion, and to their solution the rule given on page 88 (which is essentially the same as the one employed in the best schools of England and the Continent) can be applied with great facility.

The Rule for multiplying in a single line will interest by its novelty, and afford an excellent exercise for giving expertness in adding columns of two figures. It is not recommended for universal adop-

* My obligations are particularly due to SAMUEL ALSOP, Principal of Friends' Select School, and WILLIAM J. LEWIS, Teacher of the Mathematical School in Fourth street.

tion, although scholars of a decided mathematical taste will soon learn to apply it,—particularly where each factor consists of twenty or more figures,—with less liability to error than the ordinary rule.

The Algebraic symbols + and —, with a few others, have long been employed in our Arithmetics. Can any valid objection be made to the like use of letters, to represent the unknown quantity? It is certainly much more convenient to say "$7x^5$," than "7 times the 5th power of a certain number." The pupil will readily understand the application of letters, if he is told to work with them as he would with the answer to prove its correctness, and he will congratulate himself on the acquisition of a key to the many mysteries of Analysis.

Holdred's General Rule for the Extraction of Roots is universally admitted to be the best that has ever been proposed, and has therefore been substituted for the common rule, which is equally Algebraical, and much more tedious in its application.

The chapters on Divisibility of Numbers, and Numerical Approximations, will be found very useful to every one who desires to become thoroughly versed in the principles of Arithmetic. A knowledge of the theory of prime numbers (in connection with the table, which has been entirely re-calculated, and is now believed to be perfectly accurate) is of great advantage to the mathematician, and every student will find it valuable in reducing fractions to their lowest terms, and in cancelling the factors common to long multipliers and divisors.

The attention of teachers is likewise invited to the rule for dividing by 9's, the remarks on Transposition, the contractions at the close of the chapter on Practice, the Explanation of the Square Root, and the collection of valuable Tables at the close of the present volume.

The examples are of such a character as to serve the purpose of a constant review, as in each chapter the principles of the preceding chapters are involved, and the pupil is frequently led to apply all the rules with which he has become familiar.

In all respects it has been my aim to render the work practical, and at the same time rigorously mathematical. I have sought to prepare a book at once *simple*, *plain*, *concise*, *accurate* and *philosophical* in its details, and it is now submitted to the criticism of an impartial public to judge of my success.

PLINY E. CHASE.

Philadelphia, 1844.

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ARITHMETIC.

Arithmetic is the art of computing by numbers.

A Number, in Arithmetic, is an expression that denotes how many.

Written numbers may be expressed by words, by letters, or by combining the ten Arabic figures invented for this purpose:

One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Naught.
1,	2,	3,	4,	5,	6,	7,	8,	9,	0.

The first nine of these characters are called *digits*, from the Latin *digitus*, a finger: because the numbers they express are often counted on the fingers.

The tenth character, 0, expresses no value by itself, and is therefore commonly called *Naught*. It is also called *Zero*, or *Cipher*.

The number One is also called a *Unit*, and serves as a standard for comparing all numbers of a kind. Thus when we speak of ten miles, twenty houses, fourteen men, or sixty bushels, we are supposed already to have a clear idea of one mile, one house, one man, or one bushel, which is assumed as the unit.

All numbers may be considered as derived from the Unit; for one and one are two; two and one are three; three and one are four; and so on to any extent we please.

An Abstract Number, is a simple number, which has no reference to any thing in particular; as *five*, *seventeen*,

six. An APPLICATE, or CONCRETE NUMBER, is a collection of particular things; as *five books, seventeen days, six men.*

An INTEGER is a whole number; as ten, twenty-nine, forty-five. A FRACTION is a part or parts of a unit; as one-half, two-thirds, seven-tenths.

Arithmetic consists of five fundamental operations, in each of which something is given, and something is required; namely, Numeration, Addition, Subtraction, Multiplication, and Division.

In Numeration, numbers are either given in figures to be read in words, or given in words to be written in figures.

In Addition, two or more numbers are given, and their sum or amount is required.

In Subtraction, two numbers are given, and their difference is required.

In Multiplication, two numbers are given, and their product is required.

In Division, two numbers are given, and their quotient is required.

CHAPTER I.

NUMERATION.

NUMERATION is the process by which we express numbers in letters or figures, and read them when so expressed.

There are two modes of numeration now in use: the ROMAN and the ARABIC.

The Roman method employs the following letters:

One,	Five,	Ten,	Fifty,	One hundred,	Five hundred,	One thousand.
I,	V,	X,	L,	C,	D,	M.

When a letter stands before another which denotes a larger number, the value of the first must be taken from

that of the second.* In all other cases the values of the different letters must be added together, as in the following table:

I.	is	One.	XX.	is	Twenty.
II.	"	Two.	XXI, &c.	"	Twenty-one.
III.	"	Three.	XXX.	"	Thirty.
IV.	"	Four.	XL.	"	Forty.
V.	"	Five.	L.	"	Fifty.
VI.	"	Six.	LX.	"	Sixty.
VII.	"	Seven.	LXX.	"	Seventy.
VIII.	"	Eight.	LXXX.	"	Eighty.
IX.	"	Nine.	XC.	"	Ninety.
X.	"	Ten.	C.	"	One hundred.
XI.	"	Eleven.	CC, &c.	"	Two hundred
XII.	"	Twelve.	D.	"	Five hundred.
XIII.	"	Thirteen.	DC, &c.	"	Six hundred.
XIV.	"	Fourteen.	M.	"	One thousand.
XV.	"	Fifteen.	MC.	"	One thousand one hundred.
XVI.	"	Sixteen.	MDCCCC.	"	One thousand nine hundred.
XVII.	"	Seventeen.	MM.	"	Two thousand.
XVIII.	"	Eighteen.	$\overline{\text{C}}$.	"	One hundred thousand.
XIX.	"	Nineteen.	$\overline{\text{M}}$.	"	One thousand thousand.

The Arabic method is much more convenient, and is employed in all the ordinary operations of Arithmetic. By this method, *every figure is made to represent a value ten times larger by each removal to the left, and ten times smaller by each removal to the right.* Thus if 1 represent one, 10 will represent ten: 100, ten tens, or one hundred; 20, two tens, or twenty; 40, four tens, or forty; &c.

A point . called the *decimal point*, or *separatrix*, is placed at the right of units, to distinguish them from parts of units or *decimals*. The first figure at the left of the separatrix denotes units, the second tens, the third hundreds. These three are usually embraced under the name of units; the next three figures embrace units, tens, and hundreds, of thousands; the next, units, tens, and hundreds, of millions, &c., as in the following table.

---

* A dash placed over any letter, increases its value a thousand times.

Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
54	123	006	862	040	000	000	016	500	000	000	479	513	660 *

The places at the right of units are called tenths, hundredths, thousandths, &c., (denoting that the unit is divided into 10, 100, 1000, &c. parts,) as in the following table.

Trillions.	Hundred billions.	Ten billions.	Billions.	Hundred millions.	Ten millions.	Millions.	Hundred thousands.	Ten thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.	Hundred millionths.	Billionths.	Ten billionths.	Hundred billionths.	Trillionths.
6,	5	0	0,	0	5	7,	0	0	6,	9	0	8.	8	1	6	9	0	0	7	4	0	5	2	1.

In reading decimals, *read first the whole number, (if any,) then read the decimal as if it expressed a whole number, assigning it the value of the right hand decimal place.* Thus 29000001.004063, is read, 29 million and 1, and 4 thousand and sixty three *millionths.*

EXAMPLES FOR THE BOARD.

.0000090864	2987.360042
16.0003259	4.000000089443

EXAMPLES FOR THE PUPIL.

1. .08	5. .000205
2. .8	6. 13.054
3. .008	7. 208.0010076
4. .13876	8. 179002.0037

* The English formerly divided numbers into periods of six figures each, calling a million million a billion, a million billion a trillion, &c. The division into periods of three figures is now generally adopted.

9. 37000005830.0000870654
10. 2908540002.000000000000001
11. 1.00000080000070000000000000000001
12. 26501.00000000000080567809120566
13. 9003075200000854212.000000005001376
14. 116840000000000000000000000000000.05
15. 641000000873004000500926050387000.596
16. 500000000000000000005000000005000005.0005

Refer to the table of decimals, and observe that

3	decimal	places	represent	thousandths.
6	"	"	"	millionths.
9	"	"	"	billionths.
12	"	"	"	trillionths.
15	"	"	"	quadrillionths.
18	"	"	"	quintillionths.
21	"	"	"	sextillionths.
24	"	"	"	septillionths.
27	"	"	"	octillionths.
30	"	"	"	nonillionths.
33	"	"	"	decillionths.
36	"	"	"	undecillionths.
39	"	"	"	duodecillionths.

If it were required to write five thousand and eight *hundred billionths*, we should first write 5008. Billionths requiring nine decimal places, hundred billionths will require two more, or eleven. As 5008 has but four figures, seven zeroes must be prefixed to make the required number of places, thus, .00000005008. If this be numerated it will be found to express *hundred billionths*, as required.

RULE FOR WRITING DECIMALS. *Write first the whole numbers, (if any,) and put the decimal point at their right. Then write the decimals like whole numbers, and, if necessary, prefix zeroes to make up the required number of decimal places.*

EXAMPLES FOR THE BOARD.

115 Septillion, 2 trillion, 97 thousand and 5—and 49 thousand and 3 hundred thousandths.

2

Change the decimal in the above example successively to *trillionths*, *hundred quadrillionths*, and *ten sextillionths*.

19 billion 3 million 1 thousand and 16 *ten millionths* is equivalent to what whole number?

EXAMPLES TO BE WRITTEN BY THE PUPIL, AND PROVED BY NUMERATING.

1. Ten thousand and three,—and fifty-six *thousandths*.

2. Two hundred and nine,—and fifteen million and seven *trillionths*.

3. Six quadrillion seventeen billion and twenty-four,—and six quadrillion seventeen billion and twenty-four *hundred sextillionths*.

4. Seventy-five septillion three quadrillion four hundred and one thousand,—and four octillion eleven thousand and six *ten decillionths*.

5. Two quadrillion thirty trillion four hundred million and five,—and six decillion and seven *duodecillionths*.

6. One hundred duodecillion,—and one *hundred duodecillionth*.

7. One hundred and fifty sextillion fifty one quintillion and thirty-six trillion,—and six hundred and thirty billion and five *sextillionths*.

8. Twenty-eight nonillion forty-six octillion seven sextillion and nine hundred thousand,—and five hundred and twenty quintillion forty thousand and seven hundred and eight *ten septillionths*.

9. Seventeen undecillion thirty-one trillion two hundred and fifty billion,—and ninety-five thousand and thirteen *hundred quintillionths*.

10. Nine hundred and ninety-nine *undecillionths*.

11. Nine hundred,—and ninety-nine *undecillionths*.

12. Five hundred and five undecillion seventy-six nonillion and three quintillion,—and fourteen million and fifty-nine *ten octillionths*.

13. Six duodecillion twenty two quadrillion seven hundred and seventy seven *hundred duodecillionths*.

It has already been stated that zeroes have no value of themselves, their only use being to supply the place of

denominations that are wanting. ***We may therefore place as many zeroes as we please at the right of decimals, without altering their value.*** This is a truth of great importance.

CHAPTER II.

ADDITION.

ADDITION is the process by which we join two or more numbers together to find their *sum*, or *amount*.

The sign + (plus,) signifies that two or more numbers are to be added together. The sign = (equal to,) denotes that one number or series of numbers is equal to another. Thus $2+3+5=10$, signifies that the sum of 2, 3, and 5 is equal to 10.

RULE.

Write units under units, tenths under tenths, &c. Find the sum of each column, (commencing at the right hand,) and write the units' figure of the sum underneath, carrying the tens to the next column.

PROOF.

Perform the addition anew, commencing at the top and adding downwards.

FEDERAL MONEY, or the currency of the United States, is written like decimals, the dollar $ being regarded as the unit, the cents as hundredths, and the mills as thousandths.

1. Add five thousand and thirty-one dollars; two thousand eight hundred dollars and six cents; nine hundred dollars and eleven cents; five dollars sixty-two cents and five mills; and three hundred and fifty dollars and five mills.

2. Find the sum of 960840.276+28890037.0009+5613.02+2988135.921+60.

3. Find the sum of 285.9; 14.283; 1390.0025; 268; 7412.09; 3.846; 176; 506.000007; 87.284003; 441.929; and 3765.0761.

4. A steamer on the first nine days of her voyage sailed as follows: 308.02 miles; 295.0009 miles; 315.769 miles; 301.0035 miles; 256 miles; 261.4 miles; 279.0908 miles; 300.96 miles; and 298.239 miles. How far had she gone at the end of the ninth day?

5. Ten men entered into partnership, contributing as follows: $1487.638; $1960.00; $1276.45; $1195.10; $2004.375; $1703.199; $751.25; $3475.871; 11240.50; and $989.112. What amount was invested in the firm?

6. 448.771 + 2297.08 + 13596 + 67.0954 + .008876 + 14.32094 + 1559.198874 + 420 + 5995682 + .4830611 + 25700 + 619.098 + 477122.6003 = ?

7. Add together 197446.00887932; 466108.4434097; 331165; 97061588.4003; 5425980.4991844; 761716604.00491883; 26915224.20211; 8112907690080; 5176448.037165222; and 9084129765.

CHAPTER III.

SUBTRACTION.

SUBTRACTION is the process by which we take one number from another, to find their *difference.*

When one number is subtracted from another, the larger number is said to be *diminished* by the smaller. The number *to be diminished*, is the *minuend.* The number *to be subtracted*, is the *subtrahend.* The number obtained by subtraction is the *difference*, the *remainder*, or the *excess* of the larger over the smaller number.

The sign — (minus,) signifies that the latter of two numbers is to be taken from the former. Thus 7—5=2, signifies that if 5 be taken from 7, the remainder will be 2.

RULE.

Write the less number under the greater, units under units, tenths under tenths, &c. If the subtrahend has more decimal places than the minuend, annex zeroes to the latter, to supply the deficiency. Commence at the right hand and

subtract each figure from the one above it. When this cannot be done, increase the upper figure by 10 and carry 1 to the next figure you subtract.

PROOF.

The sum of the remainder and the subtrahend will be equal to the minuend.

1. What is the difference between $88463.11 and $79521.097?

2. The population of the United States in 1830 was 12858670. In 1840 it was 17068666. What was the increase in ten years?

3. A gentleman has invested $140768.25 in two estates, one of which is worth $89329.184. What is the value of the other?

4. A. said to B., I am worth $116205.393. B. said, I am not worth as much, by $49164.42. What was B. worth?

5. 8843900269517—2988143675.0087906=?

6. What is the excess of $14943870.01 over $9568841.095?

7. A broker purchased stock for which he paid $259084.638. Did he gain or lose, by selling the same stock for $290451.00? How much?

8. In a certain factory there were made 916482.7 yards of satinet, and 895267.594207 yards of broadcloth. How many yards were there of satinet more than of broadcloth?

CHAPTER IV.

MULTIPLICATION.

MULTIPLICATION is the process by which we find the sum of a number or part of a number, when repeated a given number of times.

The number *to be multiplied* or repeated, is the multiplicand. The number *to multiply by*, or the number of

times the multiplicand is repeated, is the *multiplier*. The sum obtained by multiplication, is the *product*. The multiplier and multiplicand are also called *factors* of the product. If there are more than two factors, we first obtain the product of any two, then multiply this product by a third, and so on. Thus 4 times 3 times 5 times 2=120.

Any number that can be resolved into factors, is a *composite number*. A number that cannot be so resolved, is a *prime number*. Thus 6, 12, 16, &c., are composite numbers; 1, 2, 3, 5, 7, 11, &c., are prime numbers.

The sign $\times$ (multiplied by,) denotes that one number is to be multiplied by another, as, $4\times 3=12$; $2\times 6\times 3=4\times 9$.

RULE FIRST.

Write the multiplier under the multiplicand. When the multiplier contains but one figure, commence at the right hand of the multiplicand, and multiply each figure successively, carrying the tens as in addition.

When the multiplier contains more than one figure, multiply by each figure separately, placing the first figure of each partial product under the figure by which you multiply. From the sum of all the partial products, point off as many decimal places as there are in both factors, (prefixing zeroes, if necessary, to make the required number of decimals,) and the result is the whole product.

PROOF.

Perform the multiplication anew, by RULE SECOND, *or divide the product by either factor, and the quotient will be the other factor.*

The product by zero is always zero.

To multiply any number by 10, 100, 1000, &c., *remove the decimal point as many places to the right, as there are zeroes in the multiplier.*

When there are zeroes at the right hand of either, or both factors, *perform the multiplication without them, and annex them to the product.*

If the multiplier can be resolved into factors, *the product may be obtained either by employing the whole multiplier, or each of its factors in succession.*

RULE SECOND.

Commence at the right, and number the figures of one factor, with the indices 1, 2, 3, &c., and the figures of the other factor, with the indices 0, 1, 2, 3, &c. Indicate the product of any two of the figures, by the sum of their indices. Write the units' figure of product 1, for the right hand figure of the true product, and carry the tens to THE SUM *of products 2. The units' figure of this sum is the second product figure, and the tens are carried to* THE SUM *of products 3. The units' figure of this product is the third figure sought, and the remaining figures are determined in a similar manner.*

EXAMPLE FOR THE BOARD.

Multiply in one line, 268.1475 by 93.074.

$$\begin{array}{r} {\scriptstyle 7\ 6\ 5\ 4\ 3\ 2\ 1} \\ 268.1475 \\ {\scriptstyle 4\ 3\ 2\ 1\ 0} \\ 93.074 \\ \hline {\scriptstyle 11\ 10\ 9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1} \\ 24957.5604150 \end{array}$$

According to the rule, we say $4\times5=20$. Set down 0 and carry 2. $4\times7+5\times7=63$, and 2 to carry make 65. Set down 5 and carry 6. $4\times4+7\times7+5\times0+6=71$. Set down 1 and carry 7. $4\times1+7\times4+7\times0+5\times3+7=54$. $4\times8+7\times1+4\times0+7\times3+5\times9+5=110$. $4\times6+7\times8+0\times1+3\times4+9\times7+11=166$. $4\times2+7\times6+0\times8+3\times1+9\times4+16=105$. $7\times2+0\times6+3\times8+9\times1+10=57$. $0\times2+3\times6+9\times8+5=95$. $3\times2+9\times6+9=69$. $9\times2+6=24$. This rule may be readily demonstrated, by performing the multiplication in the usual manner, and observing what figures are added together, to obtain each figure of the entire product. The multiplication in one line will be found a useful exercise, and is particularly valuable for the facility it gives in addition, as well as for the exercise of the memory.

1. $49863\times.06\times1.3=?$
2. Multiply 7441.08 by 236.
3. Find .06 of 284931.75.
4. Find .0725 of 164.25×1000.
5. $96.8\times12000\times45.93=?$
6. Multiply 4489.076 by 2604000.
7. Multiply 596000 by 70490000.
8. What is the product of $36\times24\times12\times15$?
9. A merchant borrowed $9965.73, and paid .1845 of the amount for the use of the money. How much did he pay in the whole?
10. Multiply 909871265000 by 470.0368.

CHAPTER V.

DIVISION.

Division is the process by which we find how many times one number or part of a number is contained in, or may be subtracted from, another.

The number *to be divided* is the *dividend*. The number *to divide by*, is the *divisor*. The number of times the dividend contains the divisor, is the *quotient*. The divisor and quotient may also be regarded as factors of the dividend. The number left, (if any,) after the operation, is the *remainder*.

The sign $\div$ (divided by) signifies that the former of two numbers is to be divided by the latter, as; $4\div2=2$; $16\div4=8\div2$. Division may also be expressed by writing the divisor under the dividend; as, $\frac{4}{2}$, $\frac{16}{4}$, $\frac{7}{8}$, $\frac{11}{12}$; which are read $4\div2$ or 4 *halves*; $16\div4$ or 16 *fourths*; $7\div8$ or 7 *eighths*; $11\div12$ or 11 *twelfths*, &c. Numbers written in this manner are called *fractions*, the number above the line, or the dividend, being the *numerator*, and the number below the line, or the divisor, the *denominator*. The remainder in any division may always be written as the numerator of a fraction, whose denominator will be the divisor.

RULE.

Write the divisor at the left of the dividend, and if necessary, annex decimal zeroes to the dividend, until it has as many decimal places as the divisor.

From the left of the dividend, take as many figures as will contain the divisor one or more times, for a first partial dividend. Find how many times the partial dividend will contain the divisor, and write the result as the first quotient figure. Multiply the divisor by this figure, and subtract the product from the first partial dividend. To the remainder annex the next figure of the dividend for a second partial dividend, and divide as before. Thus continue until the division is complete, and point off as many decimals in

the quotient, as there are in the dividend more than in the divisor, prefixing zeroes, if necessary, to make the required number of decimals.

PROOF.

Add the remainder to the product of the divisor by the quotient, and you will obtain the dividend.

To divide any number by 10, 100, 1000, *&c., remove the decimal point as many places to the right as there are zeroes in the divisor.*

When there are zeroes at the right hand of the divisor, cut them off, and remove the decimal point of the dividend as many places to the left. After dividing the integers, remove the decimal point from the remainder, and you will have the true remainder.

When the divisor can be resolved into factors, we may either employ the whole divisor, or each of its factors in succession.

When the quotient will contain a number of figures, it is often convenient to multiply the divisor by each of the nine digits, and write the products on the slate, before commencing the division. It will then be easy to determine the value of each quotient figure, and the product to be subtracted from the partial dividend.

EXAMPLE FOR THE BOARD.

Divide 199760 by 371.

```
371)199760(538
    1855
    ----
     1426
     1113
     ----
      3130
      2968
      ----
       162
```

The divisor is between 300 and 400. 3 hundreds are contained in 19 hundreds, 6 times, and 4 hundreds are contained 4 times. 4 and 6 are therefore the limits of the true quotient figure, which must be either 4, 5, or 6. Now the true quotient figure must be contained in 1997, at least 371 times, as its product by 371, is to be subtracted from 1997. Mentally dividing 1997 by 6, I find 6 is in 19, 3 times, 6 in 19, 3 times, which is smaller than the second figure of 371,—6 is therefore too large. Trying 5, I say, 5 in 19, 3 times, 5 in 49, 9 times, which is larger than the second figure of 371, and is therefore correct. The limits of the second quotient figure are 3 and 4. Dividing as before, 4 in 14, 3 times, 4 in 22, 5 times, which is too small. 3 is therefore the true quotient figure. The limits of the third quotient figure

are 7 and 10, or rather 7 and 9, as no figure can be greater than 9, and by mental division, 8 is found to be the true figure. Hence the following

RULE

FOR OBTAINING THE TRUE QUOTIENT FIGURE.

Employ the first figure of the divisor, and a number, one larger than the first divisor figure, as trial divisors, to determine the limits within which the true quotient figure must be found. Mentally divide the partial dividend by each of the possible quotient figures, until you find one that will give a quotient as large as, or larger than, the divisor. If there is but one such figure, it will be the one sought; if there is more than one, the larger will be the true quotient figure.

1. Divide .0497 by 368000.
2. What is the quotient of 3.995 by $2\times 3\times 4$?
3. $8.9\times .3\times 14\div 27\times .04\times 13=$?
4. If the product of two factors is 81.45, and one of the factors is 18.1, what is the other?
5. If the product of four factors is 2520, and three of its factors are 8, 5, and 7, what is the fourth?
6. Divide $4\times 7\times 9\times 2$ by $9\times 7\times 2$. By 9×4. By $4\times 2\times 9$.
7. Divide .00984 by $.02613\times 7$.
8. What is the quotient of $.087\times .003$ by 19000×70?
9. 27.3 is .16 of what number? .0095 is .84 of what number? 76.125 is 1.25 times what number? 841.21 is 1.34 of what number? 44 is .13 of what number?

THE LEAST COMMON MULTIPLE, AND THE GREATEST COMMON DIVISOR.

One number is called a *multiple* of another, when the former can be divided by the latter without any remainder. A number that can be exactly divided by two or more other numbers, is a *common multiple* of those numbers. The divisors are called *sub-multiples*, or *aliquot parts*. Thus 24 is a common multiple of 2, 4, 6, 8, and 12,—and these latter numbers are *aliquot parts* of 24. Any number that contains all the factors of a number, will evidently contain

the number itself. Thus 24, which is equal to $2\times2\times2\times3$, contains 2×2 or 4, 2×3 or 6, $2\times2\times2$ or 8, and $2\times2\times3$ or 12. Then the least common multiple of any series of numbers, is the least number which contains all the factors of the given numbers, and may be found by the following

RULE.

Arrange the numbers in a horizontal line, and divide successively by the prime numbers 2, 3, 5, 7, 11, &c., employing each divisor as often as it will divide one or more of the numbers without a remainder, writing the quotients and undivided numbers, beneath. Continue this division until the last quotients are all 1, and you will have obtained all the prime factors of the given numbers. The product of these factors, is the least common multiple.

EXAMPLE FOR THE BOARD.

Find the least common multiple of 14, 18, 27, 21, 28, 126. Dividing by the rule, we find the only prime factors contained in the given numbers are 2, 2, 3, 3, 3, and 7. Their product is 756, which is the least common multiple.

2)14 "	18 "	27 "	21 "	28 "	126
2)7 "	9 "	27 "	21 "	14 "	63
3)7 "	9 "	27 "	21 "	7 "	63
3)7 "	3 "	9 "	7 "	7 "	21
3)7 "	1 "	3 "	7 "	7 "	7
7)7 "	1 "	1 "	7 "	7 "	7
1 "	1 "	1 "	1 "	1 "	1

By the table of prime factors, the least common multiple can be found much more readily, in the following manner.

Form the product of all the prime factors of the given numbers, employing each factor the largest number of times it is used in either number.

Referring to the numbers in the foregoing example, we find they are respectively equal to 2×7, $2\times3\times3$, $3\times3\times3$, 3×7, $2\times2\times7$, and $2\times3\times3\times7$. The only prime numbers used, are 2, 3, and 7. 2 is employed 2 times in the 5th number, 3 is employed 3 times in the 3d number, and 7 is employed but once in either number. The least common multiple is then, $2\times2\times3\times3\times3\times7=756$, as before.

1. Find the least common multiple of 96, 32, 12, 24, and 48.

2. Find the least common multiple of 32, 27, 16, 18, and 54.

3. Find the least common multiple of 35, 20, 28, 21, and 12.

4. Find the least common multiple of 19, 27, 18, 12, and 36.

5. Find the least common multiple of 2, 3, 4, 5, 11, 12, and 15.

6. Find the least common multiple of 21, 25, 12, 18, and 49.

7. Find the least common multiple of 9, 11, 14, 63, and 99.

8. Find the least common multiple of 10, 6, 42, 15, 30, 105, and 210.

Any number that will exactly divide two or more other numbers, is called a *common divisor*, or *common measure*, and the greatest number that will so divide them, is the *greatest common divisor*, or *greatest common measure*, of those numbers. Thus 2, 3, and 6, are all common divisors of 6, 12, 18, 24, and 30; but their greatest common divisor is 6. Any two numbers that have no common divisor, as 6 and 7, 11 and 15, 18 and 25, are said to be *prime to each other*.

EXAMPLE FOR THE BOARD.

The common divisor of any two numbers, will also divide their difference. Thus 3 is contained in 27, 9 times, and in 36, 12 times. It must therefore be contained in 36—27, 12—9, or 3 times.

```
384)672(1
    384
    ---
    288)384(1
        288
        ---
         96)288(3
            288
```

Let it then be required to find the greatest common divisor of 384 and 672. We first divide 672 by 384, to see if the smaller number will exactly divide the larger, and we find a remainder, 288. Now the greatest common divisor of 384 and 672, is also the greatest common divisor of 288 and 384, because, if we suppose it contained 4 times in 384, and 7 times in 672, it must be contained 3 times in their difference, 288. Dividing 384 by 288, we find a remain-

der, 96. For the reason given above, the greatest common divisor sought, is also the greatest common divisor of 96 and 288. Dividing 288 by 96, we find it is contained exactly 3 times. 96 is therefore the greatest common divisor. Hence the following

RULE.

Divide the larger number by the smaller, and if there is no remainder, the smaller number will be the greatest common divisor. If there is a remainder, divide the first divisor by the first remainder, the second divisor by the second remainder, and so proceed until you obtain a quotient without a remainder. The last divisor will be the greatest common measure. If the divisor of more than two numbers is required, first find the common divisor of any two, then of this divisor and a third, and so on.

By the table of prime factors, the greatest common divisor may be found more readily in the following manner:

Form the product of the prime factors common to all the given numbers, employing each factor, the least number of times it is used in either number.

What is the greatest common divisor of 430, 602, 2150, and 3612?

By the table we find these numbers are equal, respectively, to $2\times5\times43$, $2\times7\times43$, $2\times5\times5\times43$, $2\times2\times3\times7\times43$. The only factors common to all, are $2\times43=86$, which is the greatest common divisor.

1. Find the greatest common divisor of 48, 72, and 60.
2. Find the greatest common divisor of 1001, 385, and 539.
3. Find the greatest common divisor of 405, 567, 729, and 891.
4. Find the greatest common divisor of 2863 and 1151.
5. Find the greatest common divisor of 992, 960, 928, and 32.
6. Find the greatest common divisor of 1177, 1391, and 1819.
7. Find the greatest common divisor of 2943, 2616, and 4578.
8. Find the greatest common divisor of 2148, 6444, and 3580.

CHAPTER VI.

FRACTIONS.

If we divide 16 apples among 3 boys, we can give each of them 5, and have 1 left. If we wish to divide the remaining apple among them, we must cut it into 3 equal parts, and give one to each of them. Each of those parts would be called *one-third*, and written $\frac{1}{3}$.

Again, if we wished to divide 31 apples among 9 boys, we could give each of them 3, and have 4 left. To divide those 4, we might cut each one into 9 parts, or ninths, and give each boy 4 of the parts. Then $4 \div 9 = \frac{4}{9}$.

Numbers of this kind are called *Fractions*, or broken numbers, and may be regarded in three different lights. Thus, $\frac{3}{4}$ may be read *3-fourths*, $\frac{1}{4}$ *of* 3, or 3 *divided by* 4; $\frac{31}{56}$ is 31 *fifty-sixths*, $\frac{1}{56}$ of 31, or $31 \div 56$, &c.

The dividend, or *numerator*, may be considered as numbering the parts that are taken. The divisor, or *denominator*, marks the number of parts into which a unit is divided. The numerator and denominator are also called the *terms* of the fraction.

A *proper fraction*, is one in which the numerator is less than the denominator, and the fraction is therefore less than 1, as $\frac{3}{5}$, $\frac{7}{11}$, $\frac{22}{27}$.

An *improper fraction*, is one in which the numerator is equal to, or greater than the denominator. In the former case, the fraction is equal to 1,—as $\frac{11}{11}$, $\frac{3}{3}$, $\frac{19}{19}$. In the latter, it is more than 1, as $\frac{21}{7}$, which (as $\frac{7}{7}=1$) is equal to 3; $\frac{19}{5}=3\frac{4}{5}$; $\frac{17}{2}=8\frac{1}{2}$. A whole number may always be regarded as an improper fraction, whose denominator is 1. Thus, $12=\frac{12}{1}$; $28=\frac{28}{1}$.

A *mixed number*, consists of a whole number and a fraction, as $3\frac{2}{3}$, which (as $1=\frac{3}{3}$) is equal to $\frac{11}{3}$; $126\frac{10}{13}$, which (as $1=\frac{13}{13}$) is equal to $\frac{1648}{13}$.

A *compound fraction*, is a fraction of a fraction, as $\frac{1}{4}$ of $\frac{2}{3}$; $\frac{4}{7}$ of $\frac{3}{5}$ of $\frac{7}{11}$.

A *complex fraction*, is one which contains a fraction in its numerator or denominator, as $\frac{2\frac{1}{2}}{3}$; $\frac{1}{8\frac{1}{4}}$; $\frac{3\frac{1}{2}}{4\frac{1}{3}}$; $\frac{\frac{3}{4}}{\frac{4}{5}}$.

REDUCTION OF FRACTIONS.

An improper fraction may be reduced to a whole or mixed number, *by performing the division which the fraction expresses.* Thus, $\frac{27}{11}=27\div 11=2\frac{5}{11}$. This reduction must always be made in giving the result of any operation.

A whole number may be reduced to a fraction having any given denominator, *by multiplying it by the denominator.* Thus, $2=\frac{22}{11}=\frac{26}{13}=\frac{8}{4}$.

A mixed number may be reduced to an improper fraction, *by reducing the whole number to a fraction, and adding the fraction.* Thus, since $2=\frac{22}{11}$, $2\frac{5}{11}=\frac{22}{11}+\frac{5}{11}=\frac{27}{11}$.

A compound fraction may be reduced to a simple one, *by multiplying all the numerators together for a new numerator, and all the denominators for a new denominator.* Thus, $\frac{3}{4}$ of $\frac{5}{6}=\frac{15}{24}$. For $\frac{1}{4}$ of $\frac{1}{6}=\frac{1}{24}$, because, if we suppose a unit divided into 6 equal parts, and each part again divided into 4 equal parts, the whole unit will be divided into 24 equal parts, or 24ths. Then, $\frac{1}{4}$ of $\frac{5}{6}=\frac{5}{24}$, and $\frac{3}{4}$ of $\frac{5}{6}=\frac{15}{24}$, and so of any other similar fractions. *If the numerators and denominators have common factors, they may be rejected,* as in the following example:

Reduce $\frac{1}{2}$ of $\frac{2}{7}$ of $\frac{5}{9}$ of $\frac{14}{15}$, to a simple fraction.

The fraction by the above rule, is equivalent to $\frac{1\times 2\times 5\times 14}{2\times 7\times 9\times 15}$. Rejecting the common factors 2, 7 and 5, we have $\frac{1\times 2}{9\times 3}=\frac{2}{27}$. The reason of this process will be evident, if we remember that $\frac{2\times 7\times 5}{2\times 7\times 5}=1$, and dividing any number by 1, does not alter its value. Compound fractions must always be reduced to simple ones, before performing any operation.

1. Reduce to a whole or mixed number, $\frac{149}{18}$; $\frac{270}{9}$; $\frac{759}{11}$; $\frac{421}{18}$.

2. Reduce 49 to 7ths; 12ths; 13ths; 60ths.

3. Reduce to an improper fraction, $4\frac{78}{75}$; $26\frac{1}{11}$; $149\frac{2}{3}$; $22\frac{7}{15}$.

4. Reduce to a simple fraction, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{2}{3}$ of $\frac{5}{9}$ of $\frac{12}{25}$ of $\frac{5}{8}$; $\frac{4}{7}$ of $\frac{14}{15}$ of $\frac{5}{8}$ of $\frac{2}{3}$; $\frac{3}{11}$ of $\frac{11}{14}$ of $\frac{7}{9}$ of $\frac{6}{13}$.

5. Reduce $19\frac{1}{19}$ to an improper fraction; $\frac{813}{32}$ to a mixed

number; $\frac{2376}{27}$ to a whole number; 613 to 15ths; $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{18}{19}$ of $\frac{1}{2}$ to a simple fraction.

A fraction may be reduced to a decimal, *by performing the division which the fraction expresses, annexing decimal zeroes to the numerator.* Thus, $\frac{3}{5}=3.0\div5=.6$.

A decimal may be reduced to a fraction, *by writing the decimal for a numerator, and the denomination tenth, hundredth, &c. for a denominator.* Thus $.06=\frac{6}{100}$; $.193=\frac{193}{1000}$.

A fraction may be reduced to its lowest terms, *by dividing the numerator and denominator by their greatest common divisor.* Thus $\frac{65}{117}=\frac{5}{9}$, because $\frac{65}{117}\div\frac{13}{13}=\frac{5}{9}$, and $\frac{13}{13}=1$. This reduction should always be made in giving the result of any operation. If the denominator is 10, 100, 1000, &c., *Divide both terms of the fraction by* 10, 5, *and* 2, *as often as it can be done without a remainder.* This process will evidently effect the reductions desired, because 10 contains no prime factors other than 2 and 5. In the reduction of decimals, this rule will be found the most convenient.

6. Reduce to a decimal, $\frac{7}{25}$; $\frac{3}{4}$; $\frac{1}{2}$; $\frac{8}{9}$; $\frac{7}{9}$; $\frac{6}{9}$; $\frac{5}{9}$; $\frac{4}{9}$; $\frac{3}{9}$; $\frac{2}{9}$; $\frac{1}{9}$; $\frac{5}{7}$.

7. Reduce to their lowest terms, $\frac{75}{90}$; $\frac{38}{76}$; $\frac{19}{57}$; $\frac{32}{44}$; $\frac{33}{44}$; $\frac{28}{91}$; $\frac{256}{304}$.

8. Reduce each of the following decimals to a fraction, and reduce the fraction to its lowest terms: .8 ; .014 ; .08 ; .008 ; .045 ; .1768 ; .0375 ; .25.

9. Reduce to their lowest terms, $\frac{364}{392}$; $\frac{2500}{10000}$; $\frac{765}{972}$; $\frac{345}{495}$; $\frac{3990}{5890}$.

10. Reduce to a decimal, $\frac{4}{7}$; $\frac{3}{7}$; $\frac{2}{7}$; $\frac{1}{7}$; $\frac{13}{14}$; $\frac{181}{273}$; $\frac{45}{763}$; $\frac{810}{1277}$; $\frac{463}{1804}$; $\frac{25}{3076}$.

Two or more fractions may be reduced to a common denominator, *by dividing the least common multiple of all the denominators, by each given denominator, and multiplying both terms of the fraction by the quotient.*

EXAMPLE FOR THE BOARD.

If we wish to reduce the fractions, $\frac{5}{9}$, $\frac{2}{3}$, $\frac{7}{12}$, $\frac{13}{18}$ and $\frac{5}{6}$, to a common denominator, we first find that the least common multiple is 36. Then

$36 \div 9 = 4$ and $\frac{5}{9} \times \frac{4}{4} = \frac{20}{36}$	or as there are $\frac{36}{36}$ in 1,
$\frac{1}{9} = \frac{4}{36}$ and $\frac{5}{9} = \frac{20}{36}$	
$36 \div 3 = 12$ and $\frac{2}{3} \times \frac{12}{12} = \frac{24}{36}$	$\frac{1}{3} = \frac{12}{36}$ and $\frac{2}{3} = \frac{24}{36}$
$36 \div 12 = 3$ and $\frac{7}{12} \times \frac{3}{3} = \frac{21}{36}$	$\frac{1}{12} = \frac{3}{36}$ and $\frac{7}{12} = \frac{21}{36}$
$36 \div 18 = 2$ and $\frac{13}{18} \times \frac{2}{2} = \frac{26}{36}$	$\frac{1}{18} = \frac{2}{36}$ and $\frac{13}{18} = \frac{26}{36}$
$36 \div 6 = 6$ and $\frac{5}{6} \times \frac{6}{6} = \frac{30}{36}$	$\frac{1}{6} = \frac{6}{36}$ and $\frac{5}{6} = \frac{30}{36}$

In the first mode of performing the operation, the multipliers are each equal to 1, and the product by 1 is of course equal to the multiplicand.

11. Reduce to a common denominator, $\frac{2}{3}$, $\frac{5}{6}$, $\frac{5}{12}$, $\frac{1}{2}$ and $\frac{1}{4}$.

12. Reduce to a common denominator, $\frac{6}{7}$, $\frac{3}{8}$, $\frac{5}{14}$ and $\frac{1}{4}$.

13. Reduce to a common denominator, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{5}$, $\frac{2}{3}$, $\frac{1}{2}$ of $\frac{3}{5}$ and $\frac{1}{3}$ of $\frac{2}{5}$ of 3.

The compound fractions must first be reduced to simple ones.

14. Reduce to a common denominator, $\frac{4}{5}$ of $\frac{2}{9}$, $\frac{7}{15}$, $\frac{2}{3}$ of $\frac{1}{8}$, $\frac{11}{40}$ and $\frac{1}{4}$ of $\frac{2}{3}$.

15. Reduce $29\frac{163}{164}$ to an improper fraction; $\frac{9847}{131}$ to a mixed number; $\frac{287}{1463}$ to a decimal; $\frac{798}{1037}$ to its lowest terms; $\frac{19}{36}$, $\frac{7}{54}$ and $\frac{4}{9}$ to a common denominator; .004625 to a fraction; 295 to 19ths.

ADDITION OF FRACTIONS.

The addition of Fractions having the same denominator, is as easy as the addition of any other numbers. Thus, as 5 houses + 3 houses + 2 houses = 10 houses, so $\frac{5}{7} + \frac{3}{7} + \frac{2}{7} = \frac{10}{7}$, and $\frac{5}{13} + \frac{3}{13} + \frac{2}{13} = \frac{10}{13}$. Therefore, to add fractions, *Reduce all the given fractions to a common denominator, and add their numerators.*

1. Add $2\frac{1}{3}$, $\frac{1}{2}$, $15\frac{1}{4}$, and $28\frac{5}{6}$.
2. Add $\frac{2}{3}$ of $\frac{1}{4}$, $\frac{1}{2}$ of $\frac{5}{6}$, and $19\frac{3}{4}$.
3. Add $\frac{5}{9}$, $\frac{3}{7}$, $\frac{11}{21}$, and $\frac{19}{63}$.
4. Add 19, $28\frac{1}{2}$, $16\frac{1}{11}$, and $3\frac{9}{14}$.
5. Add $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{5}$ of $\frac{6}{7}$, $2\frac{7}{10}$, $\frac{10}{21}$, and $15\frac{19}{35}$.
6. Add $12\frac{3}{4}$, $\frac{6}{7}$ of $\frac{5}{8}$, $11\frac{5}{9}$, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$, and $19\frac{1}{8}$.
7. Add $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{4}{7}$ of 9, $\frac{1}{11}$, $\frac{6}{7}$ of $\frac{2}{3}$, $\frac{14}{15}$ and $7\frac{19}{35}$.

SUBTRACTION OF FRACTIONS.

As 8 houses—3 houses=5 houses, so $\frac{8}{9}-\frac{3}{9}=\frac{5}{9}$, and $\frac{8}{14}-\frac{3}{14}=\frac{5}{14}$. The subtraction of fractions having a common denominator, is, therefore, devoid of difficulty, except when the numerator of the subtrahend is greater than that of the minuend.

EXAMPLE FOR THE BOARD.

$$\begin{array}{rr} 16\frac{2}{9} & 16\frac{11}{9} \\ 3\frac{5}{9} & 4\frac{5}{9} \\ \hline & 12\frac{6}{9} \end{array}$$

If we wish to subtract $3\frac{5}{9}$ from $16\frac{2}{9}$, we must, as in simple subtraction, increase the fraction in the minuend by 1, or $\frac{9}{9}$, and carry 1 to the units of the subtrahend. Therefore, to subtract fractions,

Reduce both fractions to a common denominator, and subtract the numerator of the subtrahend from the numerator of the minuend. If the numerator of the minuend is less than that of the subtrahend, increase it by the denominator, and carry 1 *to the units figure of the subtrahend.*

1. What is the difference between $\frac{5}{9}$ and $\frac{6}{11}$?
2. Subtract $3\frac{1}{5}$ from $11\frac{7}{8}$.
3. $1\frac{4}{7}-\frac{12}{13}=$?
4. Subtract $49\frac{5}{243}$ from $56\frac{1}{243}$.
5. From $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $7\frac{1}{8}$ take $1\frac{37}{43}$.
6. What is the difference between $4\frac{19}{87}$ and $2\frac{3}{4}$?
7. What is the difference between $\frac{1}{3}$ of $\frac{10}{11}$ and $\frac{2}{5}$ of $\frac{4}{9}$?
8. Subtract $\frac{4}{5}$ of $16\frac{1}{3}$ from $\frac{3}{8}$ of $\frac{1}{2}$ of $95\frac{2}{3}$.

MULTIPLICATION OF FRACTIONS.

If any number be multiplied by $\frac{3}{4}$, it is evident that the product will be $\frac{3}{4}$ as large as if multiplied by 1. Then $\frac{3}{4}\times 11=\frac{3}{4}$ of $\frac{11}{1}$; $\frac{2}{7}\times\frac{5}{8}=\frac{2}{7}$ of $\frac{5}{8}$. Multiplication of fractions is therefore performed in the same way as reduction of compound fractions to simple ones. Then, when either the multiplier or multiplicand is a fraction, *Change whole or mixed numbers (if any) to improper fractions, and multiply the numerators together for a new numerator, and the denominators for a new denominator.*

TO MULTIPLY A FRACTION BY A WHOLE NUMBER.—It is evident from the nature of fractions, that $\frac{1}{2}=2\times\frac{1}{4}$; $\frac{1}{3}=3\times\frac{1}{9}$; $\frac{1}{4}=4\times\frac{1}{16}$, &c. Hence, *we may multiply a fraction by a whole number, either by multiplying the nume-*

rator, or dividing the denominator. When one factor is a fraction, and the other an integer, either of these rules may be employed. When the multiplier is a mixed number, the product may often be obtained most conveniently, *by multiplying first by the whole number, and afterwards by the fraction, and adding the two products.*

1. Multiply $\frac{14}{15}$ by $3\frac{1}{9}$; $2\frac{7}{8}$ by $5\frac{1}{3}$.
2. Multiply $\frac{2}{7}$ of $\frac{8}{11}$ by $16\frac{4}{15}$.
3. Multiply $15\frac{7}{8}$ by $9\frac{1}{4}$; $28\frac{11}{13}$ by $1\frac{6}{17}$.
4. Multiply $\frac{341}{96}$ by 4; by 8; 12; 16; 24; 96.
5. Multiply $22\frac{3}{5}$ by $\frac{1}{2}$ of $\frac{4}{5}$ of $16\frac{11}{12}$.
6. $14\frac{3}{19}\times 15\frac{4}{5}=$? $8\frac{6}{7}\times 41\frac{8}{9}=$?
7. Multiply $\frac{191}{248}$ by 2; 4; 31; 8; 248.
8. Multiply $\frac{2}{13}$ of $\frac{9}{11}$ by $\frac{4}{5}$ of $\frac{3}{7}$ of $9\frac{1}{15}$.
9. What is the product of $33\frac{1}{3}$ by $4\frac{1}{2}$? by $1\frac{1}{2}$? by $15\frac{3}{4}$?

DIVISION OF FRACTIONS.

The smaller the divisor, the more times will it be contained in the dividend, and the larger will be the quotient. Thus, if $\frac{2}{3}$ be divided by 1, the quotient will be $\frac{2}{3}$. $\frac{1}{4}$ will be contained 4 times as often as 1, therefore $\frac{2}{3}\div\frac{1}{4}=\frac{4}{1}\times\frac{2}{3}$. $\frac{3}{4}$ will be contained only $\frac{1}{3}$ as often as $\frac{1}{4}$, therefore $\frac{2}{3}\div\frac{3}{4}=\frac{1}{3}$ of $\frac{4}{1}\times\frac{2}{3}=\frac{4}{3}\times\frac{2}{3}$. Then, whenever either the divisor or dividend is a fraction, *Change whole or mixed numbers (if any) to improper fractions, and if the numerators and denominators cannot be directly divided into each other, invert the divisor, and proceed as in multiplication.*

Complex Fractions, or such as contain fractions in their numerator or denominator, or both, may be resolved into simple fractions by this rule. Thus, $\frac{3\frac{5}{7}}{\frac{4}{5}}=3\frac{5}{7}\div\frac{4}{5}=\frac{26}{7}\times\frac{5}{4}=4\frac{9}{14}$.

To divide a fraction by a whole number.—It is evident that $\frac{1}{4}=\frac{1}{2}\div 2$; $\frac{1}{9}=\frac{1}{3}\div 3$; $\frac{1}{16}=\frac{1}{4}\div 4$, &c. Therefore, *We may divide a fraction by a whole number, either by dividing the numerator, or multiplying the denominator.* When the divisor is an integer, either of these rules may be employed.

1. Divide $\frac{12}{35}$ by $\frac{6}{7}$; by $\frac{3}{5}$; by $\frac{4}{35}$.

2. Divide $2\frac{11}{13}$ by $5\frac{1}{2}$; by 14; by $8\frac{2}{7}$.
3. Divide $\frac{1}{2}$ of $\frac{5}{8}$ of $\frac{3}{7}$ by $6\frac{9}{11}$; by 10.
4. Divide 4 by $11\frac{3}{16}$; by $81\frac{4}{7}$.
5. What is the quotient of $7\frac{8}{9}$ by $10\frac{11}{12}$? By 15?
6. $\frac{2}{7}$ of $\frac{2}{3}$ of $\frac{1}{8}$ of $4 \div \frac{1}{6}$ of $\frac{5}{9}$ of $13\frac{1}{2}=$?
7. Divide $\frac{91}{126}$ by $4\frac{1}{3}$; by $\frac{7}{18}$; by $3\frac{1}{2}$.
8. Divide $\frac{7}{16}$ of $19\frac{2}{7}$ by $\frac{9}{10}$ of $\frac{3}{4}$ of $8\frac{1}{2}$.
9. Divide $4\frac{12}{23}$ by $8\frac{14}{19}$; by $16\frac{1}{2}$; by $9\frac{5}{6}$.

CIRCULATING OR INFINITE DECIMALS.

In changing fractions to decimals, if the divisor contains any prime factors, other than 2 or 5, we shall always find that our work would continue without end, the same figures being repeated again and again. The decimal is then called a circulating, or infinite decimal. Thus, $\frac{1}{3}=.333+$; $\frac{2}{9}=.22222+$; $\frac{1}{11}=.090909+$; $\frac{317}{444}=.71396396+$. The figures that are thus repeated, are called the *repetend.* The figures preceding the repetend (if any) are called the *finite part* of the decimal. The repetend is usually distinguished by placing a point over the first and last figure. Thus $.\dot{3}$ is the same as $.3333+$; $\dot{0}\dot{9}$ is the same as $090909+$; $.71\dot{3}9\dot{6}$ is the same as $.71396396+$. In the latter example .71 is the *finite part*, and 396 is the repetend.

We find, by reducing the following fractions to decimals, that $\frac{1}{9}=.\dot{1}$, therefore $\frac{2}{9}=.\dot{2}$, $\frac{5}{9}=.\dot{5}$, &c. $\frac{1}{99}=.\dot{0}\dot{1}$, therefore, $\frac{2}{99}=.\dot{0}\dot{2}$, $\frac{14}{99}=.\dot{1}\dot{4}$, $\frac{95}{99}=.\dot{9}\dot{5}$, &c. $\frac{1}{999}=.\dot{0}0\dot{1}$, therefore, $\frac{2}{999}=.\dot{0}0\dot{2}$, $\frac{45}{999}=.\dot{0}4\dot{5}$, $\frac{163}{999}=.\dot{1}6\dot{3}$, &c.

Hence, *every repetend is equivalent to a fraction, having the repetend for its numerator, and an equal number of 9's for its denominator.*

TO REDUCE INFINITE DECIMALS TO FRACTIONS.—We have seen that $71.\dot{3}4\dot{2}=71\frac{342}{999}=\frac{71271}{999}$. Therefore, $.071\dot{3}4\dot{2}$ (which is $\frac{1}{1000}$ of $71.\dot{3}4\dot{2}$),$=\frac{71271}{999000}$. In this example, the numerator is equivalent to $71342-71$; that is, to the whole decimal, *minus* its finite part. The denominator contains as many 9's as there are figures in the repetend, with as many 0's as there are finite figures.

RULE.

If the repetend commences at tenths, consider the repetend as the numerator, and an equal number of 9's as the denominator. If there are integral figures in the repetend, annex as many zeroes to the numerator. If the decimal has a finite part, from the decimal, considered as a whole number, subtract the finite part for a numerator. For a denominator, write as many 9's as there are figures in the repetend, annexing as many zeroes as there are finite figures.

1. Reduce $.\dot{0}1\dot{8}$ to a fraction, and reduce the fraction to its lowest terms.

2. Reduce each of the following numbers to a fraction: $\dot{2}.\dot{7}$; $\dot{8}9.\dot{1}$; $.\dot{0}0\dot{4}$; $.\dot{0}51\dot{3}$; $\dot{6}.64\dot{9}$.

3. Reduce each of the following numbers to a fraction: $.16\dot{3}9\dot{4}$; $.852\dot{9}0\dot{6}$; $\dot{2}45.\dot{9}$; $06\dot{8}4\dot{3}$.

Addition, Subtraction, Multiplication and Division of Infinite Decimals can usually be performed with sufficient accuracy, by extending the decimals to six or eight figures each. In Multiplication and Division, if an exact result is required, we must first change the decimals to fractions.

1. Add $96.\dot{0}2\dot{4}$, $3\dot{7}.8\dot{1}$, $495.5\dot{7}\dot{6}$, and 13.3.
2. From $\dot{4}7.\dot{8}$, subtract $19.\dot{4}\dot{3}$.
3. Subtract $15.06\dot{4}$ from $329.\dot{8}$.
4. $14.2\dot{4}\dot{9}+.\dot{6}8\dot{3}+4.90\dot{7}-17.\dot{6}38\dot{4}=$?
5. Multiply 16.947 by $.21\dot{8}\dot{4}$.
6. Multiply $29.\dot{3}$ by $1.\dot{6}\dot{7}$
7. Divide $.\dot{3}8\dot{3}$ by $\dot{9}.\dot{7}$; $.\dot{4}$ by $\dot{6}\dot{1}$.

RULE FOR DIVIDING BY 9's.

When the divisor consists of any number of 9's, increase it by 1, for a new divisor. Divide the dividend by this new divisor. By the same divisor, divide the integers of the quotient, and proceed in a similar manner, until a quotient is obtained less than the divisor. Add all the quotients together, observing the number of units carried from decimals to integers. Add this carriage to the right hand decimal figure, and the integers will

represent the quotient, and the decimals the remainder. When all but the units' figure of the divisor are 9's, increase the divisor by the difference between the units figure and 10, and divide as above directed, multiplying each quotient after the first, by the number added to the divisor. Multiply the number carried from decimals to integers, by the number added to the divisor, and add the product to the decimals for the true remainder. If this increased remainder exceeds the divisor, increase the quotient by 1, and subtract the divisor from the remainder for the true remainder.

EXAMPLE FOR THE BOARD.

```
8905.473
   8.905
       8
--------
8914.386
       1
8914.387 quotient.
```

Divide 8905473 by 999. The divisor increased by 1, is 1000. Dividing by the rule, and adding the quotients, we obtain 8914.386. There being 1 unit to carry from decimals, we add 1 to the right hand decimal figure, and find the quotient is $8914.\dot{3}8\dot{7}$.

This rule is founded on the decimal value of $\frac{1}{999}$, and it will be easily seen that the process is nearly the same as in the multiplication by $.00\dot{1}00\dot{1}+$. In dividing the numerators of fractions obtained by the multiplication of circulating decimals, the rule will often be of use.

```
154963.8144
   139.4667
      1251
-----------
155103.4062
   1×9=9
-----------
      .4071 remainder.
```

Divide 1549638144 by 9991. The divisor increased by 9, is 10000. Dividing first by this number, we multiply the integers of the first quotient by 9, writing the first figure of the product under the right hand decimal figure, which is equivalent to multiplying by 9, and dividing by 10000. We multiply the integers of this second number, and write them in the same manner, and add the several numbers together. There being 1 unit to carry from decimals, we multiply it by 9, and add the product to 4062, which gives 4071 for the true remainder.

1. Divide 870468 by 999; by 99.

2. Divide 408176 by 999; by 9999.

3. Divide 7963142 by 999000. Divide first by 999, then divide the quotient by 1000.

4. Divide 1046.84 by 993; by 9991.

5. Divide 87364001 by 99999.

6. Divide 23719.504 by 999; by 995.

7. Multiply $6.4\dot{9}1\dot{5}$ by 8.3.

8. Multiply $.4\dot{1}7\dot{2}$ by .916.

CHAPTER VII.

COMPOUND NUMBERS.

When several denominations of the same kind (as pounds, shillings and pence; miles, feet and inches; gallons, quarts and pints) are embraced in one sum, they are called COMPOUND NUMBERS. The operations upon them, are performed by regarding each denomination as a fraction of the next higher.

REDUCTION OF COMPOUND NUMBERS.

EXAMPLE FOR THE BOARD.

r.	*yd.*	*ft.*	*in.*
27	" 4	" 2	" 11

```
 5.5
 ———
 135
135
——————
148.5 yd.
  4   yd.
——————
152.5 yd.
    3
——————
457.5 ft.
  2   ft.
——————
459.5 ft.
   12
——————
5514.0 in.
  11   in.
——————
5525   in.
```

How many inches in 27 *r.* 4 *yd.* 2 *ft.* 11 *in.*? As there are 5.5 *yards* in 1 *rod*, in 27 *rods* there will be 148.5 *yds.* In 27 *r.* 4 *yd.* there will then be 152.5 *yd.* As 3 *ft.* make 1 *yd.*, in 152.5 *yd.* there will be 457.5 *ft.* Adding the 2 *ft.* we obtain 459.5 *ft.* As 12 *in.* make 1 *ft.*, in 459.5 *ft.* there are 5514 *in.* Adding the 11 *in.* we have 5525 *in.* for the answer. Hence, to reduce the higher denominations of a compound number to their value in a lower denomination, we have the following rule.

RULE.

Commence with the highest denomination, and multiply each denomination by the number required of the next lower to compose it, adding to the product the number (if any) already in that denomination. Thus proceed until you have reached the lowest denomination sought.

```
12)5723
   ————
  3)476 ft. " 11 in.
    ———
5.5)158 yd. "  2 ft.
    ———
     28 r.  "  4 yd.
```

How many *r. yd. ft.* and *in.* in 5723 inches? 5723 *in.*$=\frac{5723}{12}$ *ft.*=476 *ft.* 11 *in.* 476 *ft.*$=\frac{476}{3}$ *yd.*=158 *yd.* 2 *ft.* 158 *yd.*$=\frac{158.0}{5.5}$ *r.*=28 *r.* 4 *yd.* The answer is, therefore, 28 *r.* 4 *yd.* 2 *ft.* 11 *in.* Hence, to reduce the lower denominations of a compound number to their value in higher denominations, we have the following

RULE.

Divide each denomination by the number required of it to make 1 of the next higher, and give to each remainder the name of the dividend from which it is derived.

1. In 41 *chal.* 19 *bu.* of coal, how many pecks?
2. How many *cwt.* &c., in 1870953 *gr.*?
3. How many ℔, &c., in 24753 *gr.* Troy?
4. Reduce 195℔ 11℥ 3ʒ to *grains.*
5. Reduce 69741895 *inches* to *miles,* &c.
6. Reduce $\frac{7}{16}$ *cwt.* to *qr. lb.* &c.*
7. Reduce £9.481 to £ *s.* &c.
8. Reduce 7964 gills to gallons, &c.
9. Reduce $\frac{49}{63}$ *Y.* to days, &c.
10. Reduce $51\frac{3}{11}$ *Cong.* to *Cong. O.*, &c.
11. Reduce 8753 *in.* to *yd. qr.*, &c.
12. How many acres in a piece of land that is 15*r.* 4*yd.* long, and 14 *r.* 2 *yd.* 2 *ft.* wide?
13. How many cords in a pile of wood 21.5*ft.* long, 8.3*ft.* wide, and 9.7*ft.* high?
14. Reduce $4\frac{2}{9}$ *Y.* to seconds.

---

* The reduction of fractions and decimals is performed in the same way as reduction of whole numbers, by multiplying or dividing, as the question may require.

15. How many tons of hewn timber in 19 sticks, each stick being 18.5*ft.* long, 1.3*ft.* wide, and 1.1*ft.* thick?

16. How many drops of distilled water would fill a cistern that is 6*ft.* 8*in.* long, 5*ft.* 3*in.* wide, and 4*ft.* 2*in.* deep? [First find the number of cubic inches in the cistern, then divide by 231 (because 231 cubic in. make a Cong.), and multiply by the number of drops in a Cong.]

17. What is the value of £197 7*s.* 6*d.* in Federal Money, estimating the penny at 2 cents?

18. What is the value of 6971 roubles 25 copecks, at $\frac{3}{4}$ of a cent per copeck?

19. Reduce $\frac{7}{11}$ of a mile to *fur.*, *r.* &c.

20. Reduce 10 pounds 17 shillings 10 groats Flemish, to dollars and cents, at a cent a groat.

21. Reduce 172843'''' to ft., primes, &c.

22. The specific gravity* of gold is 19.258. Then, what is the weight of a mass of gold that is 6 inches long, 5 inches wide, and 3 inches thick?

TO REDUCE LOWER DENOMINATIONS TO THE FRACTION OF A HIGHER.

What part of a £ is 2*s.* 7*d.* 3*qr.*?

2*s.* 7*d.* 3*qr.* $=127qr.=\frac{127}{4}d.=\frac{127}{4}$ of $\frac{1}{12}s.=\frac{127}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ £ $=\frac{127}{960}$£.

RULE.

First reduce the given sum to the lowest denomination mentioned. With this result make a fraction of the next higher denomination, then a compound fraction of the next higher, and so proceed until you reach the denomination required.

1. Reduce 9*s.* 3*d.* 2*qr.* to the fraction of a £.
2. Reduce 3*qt.* 1*pt.* 2*gi.* to the fraction of a gallon.
3. Reduce 13*h.* 11*min.* 11*sec.* to the fraction of a day.
4. Reduce 4*yd.* 10*in.* to the fraction of a rod.
5. Reduce 7*O.* 6*f*℥, to the fraction of a gallon.
6. Reduce 7*s.* 6*d.* to the fraction of a guinea.

* The specific gravity of a body, is its weight compared with an equal bulk of water. Thus, gold being 19.258 times as heavy as water, its specific gravity is 19.258.

7. Reduce 3*qr.* 3*na.* to the fraction of a yard.

8. What part of 7*lb.* is 13*oz.* 14*dr.*?

9. What part of 576 grammes is 43 centigrammes?

10. Reduce 6*sq. ft.* 130*sq. in.* to the fraction of a rood.

TO REDUCE LOWER DENOMINATIONS TO THE DECIMAL OF A HIGHER.

Reduce 10*oz.* 15*dwt.* 22*gr.* to the decimal of a ℔. 22*gr.*$=\frac{22}{24}$ *dwt.*$=.91\dot{6}$*dwt.* Therefore, 15*dwt.* 22*gr.*$=15.91\dot{6}$*dwt.*$=\frac{15.91\dot{6}}{20}$*oz.* $=.7958\dot{3}$*oz.* Then, 10*oz.* 15*dwt.* 22*gr.*$=10.7958\dot{3}$*oz.*$=\frac{10.7958\dot{3}}{12}$℔. $=.899652\dot{7}$*lb.*

RULE.

Commence with the lowest denomination, annex decimal 0's, and reduce each denomination to the decimal of the next higher, by dividing as in ordinary reduction.

In the following examples, the numbers are all to be reduced to the decimal of the highest denomination mentioned.

1. Add £11 11*s.* 11*d.*; £4 3*s.* 7*d.*; 2*s.* 6*d.* 3*qr.*; and 9*s.* 6*d.*

2. Add 6*gal.* 3*qt.* 1*pt.* 3*gi.*; 2*qt.* 1*pt.* 1*gi.*; 5*gal.* 3*gi.*; and 4*gal.* 1*qt.*

3. From 19*T.* 10*cwt.* subtract 15*cwt.* 3*qr.* 14*lb.*

4. From 7*Cong.* 7*O.* subtract 1*Cong.* 7*f*℥.

5. Multiply 14*Y.* 36*d.* 12*h.* by 9.4.

6. Divide 17℔ 6℥ 4ʒ by 5.

7. Divide 29*r.* 4*yd.* 2*ft.* by 3*yd.* 1*ft.* 6*in.*

8. Reduce £591 9*s.* 3*d.* to Federal Money, at $4.85 per £.

9. Reduce 28 ducats 9 carlins 7 grains of Naples, to dollars and cents, at 83*cts.* per ducat.

10. Reduce 28*C.* 7*C. ft.* 4*c. ft.* to cords and decimals of a cord.

11. What is the value of 94 francs 79 centimes, at 18$\frac{3}{4}$ cts. per franc?

12. What is the value of 75 milrees 2 crusades, at $1.15 per milree?

COMPOUND ADDITION.

EXAMPLE FOR THE BOARD.

£.	*s.*	*d.*	*qr.*
20	16	9	3
11	0	7	1
9	19	11	2
41	17	4	2

What is the sum of £20 16*s.* 9*d.* 3*qr.*; £11 7*d.* 1*qr.*; and £9 19*s.* 11*d.* 2*qr.*? Writing the numbers as in Simple Addition, placing those of the same denomination in the same perpendicular column, I commence with the column of farthings, and find its sum, 6*qr.*=1*d.* 2*qr.* I write the 2*qr.* beneath the column of *qr.*, and carry the 1*d.* to the column of *d.* The sum of the column of *d.* is 28*d.*=2*s.* 4*d.* Write the 4*d.* beneath the column of *d.* and carry 2*s.* to the column of *s.* There are 37*s.*=£1 17*s.* Write 17 beneath the column of *s.* and carry £1 to the column of £. The sum of the £ is 41, which is written down, as there is no higher denomination.

RULE.

Write the numbers as in Simple Addition, placing those of the same denomination in the same vertical column. Commence at the right hand to add, and reduce the sum of each column to the next higher denomination. Write the remainder underneath, and carry the quotient to the next column.

1. Add 9*Qr.* 1*C.* 2*bu.* 3*pk*; 4*Qr.* 1*bu.* 1*pk.* 1*gal.*; 3*bu.* 3*pk.* 1*qt.*; and 5*Qr.* 2*pk.*, and reduce the result to quarts.

2. Add 3*Cong.* 7*O.* 15*f*℥ 7*f*ʒ 59♏.; 7*Cong.* 3*f*℥; 4*O.* 9*f*℥ 4*f*ʒ; and 2*Cong.* 4*f*ʒ 1 ♏, and find the weight of the whole.

3. Add 116*gal.* 3*qt.* 1*pt.*; 59*gal.* 1*pt.* 3*gi.*; 76*gal.* 2*qt.* and 291*gal.* 1*qt.* 1*pt.* 1*gi.*, and reduce the result to gallons and decimals of a gallon.

4. Add 14*T.* 9*cwt.* 3*qr.* 17*lb.*; 9*T.* 1*qr.* 25*lb.* 5*oz.*; 16*lb.* 3*oz.* 14*dr.*; and 17*T.* 18*cwt.* 1*qr.* 27*lb.* 2*dr.*

5. Add £19 7*s.* 6*d.* 3*qr.*; £276 19*s.* 11*d.*; 14*s.* 3*qr.*; £278 13*s.*; and £190 5*s.* 5*d.* 2*qr.*

6. Add 5℔. 11℥ 7ʒ 2℈ 18*gr.*; 7℔. 14*gr*; 3℥ 3ʒ; and 15℔. 4℥ 19*gr.* and reduce the ℥, &c., to the fraction of a ℔.

7. Add 55*yd.* 3*qr.* 3*na.*; 7*yd.* 1*na.* 1 *in.*; 4*yd.* 2*qr.* 1*na.*; 63*yd.* 2*na.*; and 175*yd.* 1*qr.* 2*na.* 2*in.*, and reduce the result to decimal yards.

8. Add 3*C.* 7*C.ft.* 15*c.ft.* 1719*c.in.*; 14*C.* 3*c.ft.* 96*c.in.*; 2*C.ft.* 14*c.ft.*; 47*C.* 6*C.ft.*; and 19*C.*

9. Add 9*sq. m.* 627*A.* 3*R.* 30*r.*; 3*sq. m.* 248*A.* 39*r.*; 41*sq. m.* 27*r.*; and 163*A.* 2*R.* 12*r.* and reduce the result to rods.

10. Add 17*m.* 7*fur.* 39*r.* 4*yd.* 2*ft.* 11*in.*; 19*m.* 3*fur.* 3*yd.*; 6*fur.* 18*r.* 1*ft.* 9*in.*; and 41*m.* 24*r.* 2*ft.* 7*in.*

11. How many deniers in 4 Louis d'ors 1 ecu 5 livres +13 Louis 2 livres 19 sous 10 deniers+2 livres 7 sous+4 livres 13 sous 9 deniers?

12. Add 19 ducats 8 carlins 7 grains; 12*d.* 3*c.* 5*gr.*; 9*c.* 8*gr.*; and 29*d.* 2*c.* and find their value at 81 cts. per ducat.

13. Add 19*ft.* 11′ 8″ 5‴; 3*ft.* 4″ 9‴; 10′ 10″; 8*ft.* 6′; 15*ft.*; and 29*ft.* 2′ 7″ 11‴.

COMPOUND SUBTRACTION.

EXAMPLE FOR THE BOARD.

bu.	*pk.*	*qt.*	*pt.*
39	" 1	" 3	" 1
19	" 2	" 5	" 1
19	" 2	" 6	" 0

From 39*bu.* 1*pk.* 3*qt.* 1*pt.* take 19*bu.* 2*pk.* 5*qt.* 1*pt.* Commencing at the right hand, as in Simple Subtraction, I say 1*pt.* from 1*pt.* leaves 0*pt.* 5*qt.* cannot be taken from 3*qt.*, I therefore increase both numbers by 1*pk.* or 8*qt.* and say, 5*qt.* from 11*qt.* leaves 6*qt.*, 3*pk.* from 5*pk.* leaves 2*pk.*, and 20*bu.* from 39*bu.* leaves 19*bu.*

RULE.

Write the numbers as in Compound Addition, and commence at the right hand to subtract. If the number representing any denomination in the minuend, is less than that in the subtrahend, increase each by one of the next higher denomination, and proceed as in ordinary subtraction.

1. From 28*chal.* 3*bu.* 1*pk.* take 18*chal.* 31*bu.* 2*pk.* 7*qt.*

2. What is the difference between 187*gal.* 3*qt.* 1*pt.* 3*gi.* and 216*gal.* 2*gi.*?

3. Subtract 7*Y.* 191*d.* 18*h.* 59*min.* 2*sec.* from 8*Y.* 32*d.* 17*h.*

4. From 19*Cong.* subtract 7*O.* 11*f*℥ 3*f*ʒ 41♏, and reduce the remainder to ♏.

5. A grocer purchased 19*cwt.* 1*qr.* of sugar. How much had he left after selling 11*cwt.* 2*qr.* 17*lb.* 12*oz.*?

6. A man sold his horse for £40, which was £9 8*s.* 6*d.* more than it cost him. What did he give for the horse?

7. Subtract 9℔ 7℥ 7ʒ 1℈ 11*gr.* from 16℔ 3ʒ 9*gr.*

8. Subtract 19*yd.* 3*qr.* 3*na.* from 117*yd.* 1*qr.* 2*na.*

9. A ship left port Nov. 29th, 1841, and returned Aug. 18th, 1844. How long was her voyage?

10. From a lot of land containing 490 Acres, 163A. 3*R.* 17*r.* were sold. How much was left?

11. From a board measuring 13*ft.* 9′ 11″, a piece was sawed containing 4*ft.* 0′ 3″ 8‴. How much was left?

12. How many seconds elapsed from 20*min.* past 5 A. M. 8th mo. 18, 1829, to 17*min.* past 3 P. M. 5th mo. 3, 1831?

13. Subtract 9 marcs 13 schillings 11 pfennings Hamburg, from 10*m.* 3*sch.*, and reduce the remainder to the fraction of a marc.

14. From 7*m.* 3*fur.* 9*ch.*, subtract 7*fur.* 3*p.* 18*l.*, and reduce the remainder to miles and decimals of a mile.

COMPOUND MULTIPLICATION.

EXAMPLE FOR THE BOARD.

yd.	*ft.*	*in.*
9 "	2 "	11
		15
149 "	1 "	9

Multiply 9*yd.* 2*ft.* 11*in.* by 15. Multiplying the inches by 15, we find 165*in.*=13*ft.* 9*in.* Write 9*in.* below the line, and carry 13*ft.* 15 times 2*ft.*=30*ft.* and 13*ft.* to carry, make 43*ft.* =14*yd.* 1*ft.* Write 1*ft.* in the column of feet, and carry 14*yd.* 15 times 9*yd.*=135*yd.* and 14*yd.* to carry, make 149*yd.*

RULE.

Commence with the smallest denomination, and reduce each product to the next higher denomination, writing the remainder underneath, and carrying the quotient as in Compound Addition.

1. Multiply 13*Y.* 79*d.* 18*h.* 43*m.* 12*s.* by 43.

2. Multiply 19*m.* 7*fur.* 4*r.* 4*yd.* 11*in.* by 81.

3. Multiply £96 11*s.* 11*d.* 1*qr.* by 57.

4. Multiply 41*yd.* 2*qr.* 3*na.* 1*in.* by 79.

5. Multiply 28*T.* 18*cwt.* 2*qr.* 13*lb.* by 47.

6. Multiply 781*gal.* 1*gi.* by 46.

7. Multiply 83 *Cong.* 7 *f*ʒ 41 ♏ by 23.

8. How many acres, roods, &c., in 73 pieces of land, each of which is 73.8 rods long, and 31.3 rods wide?

9. How many cords, &c., of wood, in 61 piles, each pile being 12.5*ft.* long, 4*ft.* wide, and 8.75*ft.* high?

10. How many seconds in 13 times 4*Y.* 18*d.* 13*h.*?

11. How many feet, primes, &c., in 86 times 3 9′ 0″ 4‴?

12. Multiply 7*m.* 3*fur.* 5*ch.* by 17, and reduce the *fur.* and *ch.* to the fraction of a mile.

13. Multiply 19 4′ 2″ 11‴ by 29, and reduce the result to ‴.

14. Add 425*lb.* 13*oz.* 15*dr.*; 37*lb.* 14*oz.*; 91*lb.* 2*oz.* 11 *dr.*;—subtract 500*lb.* 15*oz.* 14*dr.* 2*sc.* from the sum;—multiply the remainder by 11, and reduce the result to grains.

COMPOUND DIVISION.

EXAMPLE FOR THE BOARD.

yd.	*ft.*	*in.*
5)19 "	2 "	9
3 "	2 "	$11\frac{2}{5}$

Divide 19*yd.* 2*ft.* 9*in.* by 5. Dividing the 19*yd.* we have a quotient of 3*yd.* with a remainder, 4*yd.* The 4*yd.* undivided, are equivalent to 12*ft.*, which, added to the 2*ft.* make 14*ft.* 14*ft.*÷5=2*ft.* with a remainder, 4*ft.* The 4*ft.* undivided, are equal to 48 inches, which, added to the 9 inches, make 57 inches. 57 *in.*÷5=$11\frac{2}{5}$ *in.*

RULE.

First divide the highest denomination. Whenever there is any remainder, reduce it to the next lower denomination, and add it to the number (if any) already in that denomination. Then divide as before, and so proceed until the division is completed. If both the divisor and the dividend are compound numbers, reduce them to the same denomination, and divide as in simple division.

1. What is the price of a *qr.* of sugar, at £2 3*s.* per *cwt.*?

2. Divide 93*gal.* 1*qt.* by 11 ; by 19.

3. How many barrels, each holding 2*bu.* 3*qt.* can be filled with 41*bu.* 2*pk.* of apples ?

4. Divide 119*Cong.* 15*f*℥ 3*f*ʒ by 28.

5. Divide 79*yd.* 1*qr.* 1*na.* by 4*yd.* 3*qr.*

6. Divide $\frac{7}{8}$ of a mile by 3*ft.* 9*in.*

7. Divide 4.76*Y.* by 3.9*d.*

8. Divide 196℔ 11℥ 2℈ by 24.

9. Divide $27\frac{5}{7}$*T.* by $\frac{3}{7}$*cwt.*

10. How many house lots, each containing 1*A.* 1*R.* 15*r.* can be made from a piece of land 219*r.* long, and 63*r.* wide ?

11. Divide 37 6′ 6″ by 9; by 13.

12. Divide 49×£6.725 by 31 ; by 53.

13. In $\frac{1}{63}$ of 287*yd.* 3*qr.* 2*na.* 2*in.* how many yards and decimals of a *yd.* ?

14. What is the value of $\frac{1}{59}$ of 7631 piastres 35 paras, at 8 cents a piastre ?

15. Subtract 91 francs 63 centimes from 1720*fr.* 11*c.*, divide the remainder by 81, and find the value of the quotient, at 18*cts.* a franc.

DUODECIMALS.

Duodecimals are compound numbers, in which each denomination is $\frac{1}{12}$ of the one above it. They are used in square and cubic measure.

The denominations of Duodecimals are, the foot, or unit, the inch, or prime, marked (′), the second, or twelfth of a prime (″), the third, or twelfth of a second (‴), the fourth (⁗), and so on. These marks are called *indices.*

Duodecimals are added and subtracted in the same manner as other compound numbers, but multiplication and division, when both numbers contain duodecimals, present some difficulties.

The product of units by units is units ; units by primes give primes, units by seconds give seconds, &c. Primes by primes give seconds ; primes by seconds give thirds, &c. Generally, to the product of any two denominations,

we must annex as many indices as there are in both factors. For example, $5'' \times 5''''$ is the same as $\frac{5}{12}$ of $\frac{1}{12} \times \frac{5}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$. In one factor, the denominator 12 is contained twice, in the other, four times; then in their product it will be contained six times, which will be represented by $25''''''$. For a similar reason, in division we subtract the indices of the divisor from those of the dividend. Then $32'''' \div 8'' = 4''$.

EXAMPLES FOR THE BOARD.

2 "	3′	7″				
		9 "	5′	11″		
20 "	8′	3″				
	11′	5″	11‴			
	2′	1″	3‴	5⁗		
21 "	9′	10″	2‴	5⁗		

What is the product of 2 3′ 7″ by 9 5′ 11″? We write the units' figure of the multiplier under the right-hand figure of the multiplicand, because the product of any denomination by units is of the same value. The product by 12ths will be 12 times smaller than that by units, the product by seconds, 12 times smaller than that by primes, and so on. We therefore write the first figure of each product under the figure by which we multiply, as in simple multiplication. When any product, or the sum of any column, is greater than 12, we reduce it to its value in higher denominations, and carry the quotient to the next column at the left.

Divide 21 " 9′ 10″ 2‴ 5⁗ by 9 " 5′ 11″.

```
9 " 5′ 11″) 21 " 9′ 10″ 2‴ 5⁗ (2 " 3′ 7″
            18 " 11′ 10″
            ------------
             2 " 10′  0″ 2‴
             2 "  4′  5″ 9‴
             --------------
                  5′  6″ 5‴ 5⁗
                  5′  6″ 5‴ 5⁗
                  ------------
```

9 (units) is contained in 21 (units) 2 (units) times.—Multiplying and subtracting, we have a remainder 2 " 10′ 0″. Bringing down the 2‴ we reduce 2 " 10′ to primes, and find 34′ contain 9, 3′ times. Multiplying and subtracting as before, we have a remainder 5′ 6″ 5‴, to which we annex the 5⁗ of the dividend. 5′ 6″=66″ which contain 9, 7″ times. Multiplying by 7″, there is no remainder; the quotient is therefore, 2 " 3′ 7″. These operations are so analogous to those of Simple Multiplication and Division, that they require no separate rules.

1. How many feet, &c., are there in 6 boards, each of which is 13 11′ long, and 1 9′ wide?

2. How many yards of carpeting that is 2 10′ wide, will cover a floor 16 feet long and 13 6′ wide?

3. A carpenter has four boards, the first of which is 12*ft.* long and 2*ft.* wide; the second, 11 9′ long and 1 9′ wide; the third, 12 7′ long and 1 11′ wide, and the fourth, 13 2′ long and 1 8′ wide. How many square feet are there in the whole?

4. How many cubic feet in a wall 19*ft.* 6*in.* long, 2*ft.* 3*in.* thick, and 4*ft.* 6*in.* high?

5. How many cubic feet of wood in a pile 16 3′ long, 4 6′ wide, and 8 feet high?

6. The number of square feet, &c., in a floor, is 166 9′ 6″ 3‴ 9⁗, and the length of the room is 17 11′ 3″. What is the breadth?

7. What is the price of a pile of wood 128*ft.* long, 8 9′ wide, and 6 3′ high, at $6.25 per cord?

8. How much plastering is there in the ceiling of a room 21 4′ long and 18 7′ 6″ wide?

9. How many square feet of glass in 8 windows, of 12 panes each, each pane being 1 1′ long and 9′ wide?

10. Add 4 0′ 3″, 18 5′, and 3 8′ 2″, subtract from the sum 13 5′ 11″, multiply the remainder by 5 times 4 2′ 4″, and divide the product by 4 2′ 6″.

MISCELLANEOUS EXAMPLES.

1. $316\frac{7}{8}+24.59+119\frac{2}{3}-78\frac{4}{15}\times 12\frac{8}{11}\div 96\frac{1}{6}=$?

2. In a certain farm that is 142.5 rods long, and $119\frac{3}{4}$ rods wide, there is a pond containing $13\frac{2}{7}$ acres. How many acres are not covered by water?

3. The pyramid of Cheops measures 763.4 feet on each side of its base. How many acres does it cover?

4. A farmer purchased 49*A.* 3*R.* 31*r.* of woodland, $56\frac{5}{8}$*A.* of pasture, 119*A.* 27*r.* of meadow, and $38\frac{2}{7}$*A.* of tillage. What did he give for the whole, at $28.50 per acre?

5. A mason builds a wall 14 11′ 3″ long, 2*ft.* thick, and 4 6′ high. What will he receive for the whole, at $0.75 per foot?

6. What would be the weight of a gold wire that would

reach from Boston to Liverpool, estimating the distance at 2997*m.* 3*fur.* 28*r.*, and the weight of the wire at $\frac{1}{1800}$*gr.* avoirdupois, per foot?

7. The wheel of a carriage measures 13.625 feet in circumference. How many times will it revolve in going $96\frac{2}{3}$ miles?

8. How many Wine gallons will fill a cubical box that is 9.7 feet long, 4*ft.* 5*in.* wide, and $8\frac{2}{3}$ feet high? How many Imperial gallons?

9. What is the cost of 9*T.* 11*cwt.* 3*qr.* of sugar, at 5*d.* 3*qr.* per pound?

10. How many bottles, each holding 1*O.* 14*f℥.* 28♏, may be filled with 29*Cong.* of mixture, and how much will be left?

11. What part of £279 11*s.* is £31 9*s.* $6\frac{3}{4}$*d.*?

12. How many bushels of potatoes at 2*s.* 6*d.* per bushel, will pay for $49\frac{7}{8}$*yd.* of cloth, at £$1\frac{1}{7}$ per yard?

13. How many times will a seconds' pendulum tick in 3*Y.* 19*d.* 4*h.*?

14. What must I pay for $3\frac{7}{8}$*yd.* of cloth, if $2\frac{5}{9}$ *yd.* cost $43\frac{3}{8}$ cents?

15. I have a tea-spoon that holds 28 drops of water. How many times must it be filled to give me 1*lb.* 12*oz.* of water?

16. When wood is $5.375 per cord, what must I pay for 19 loads, each measuring 8.3*ft.* long, 3.5*ft.* wide, and 4.25*ft.* high?

17. Divide .019 of .00386 by $\frac{7}{8}$ of 269.03.

18. A grocer purchased $19\frac{3}{4}$*cwt.* of sugar, at $6\frac{1}{2}$ cents a pound, and sold the whole for $156.75. How much did he gain by the sale?

19. Bought 95*yd.* 3*qr.* 2*na.* of broadcloth, at $3.50 per yard. For how much must I sell the whole to gain $35.75?

20. What is the least common multiple of 130, 546, 156, and 182?

21. What is the greatest common divisor of 111, 222, 333, 444, 555, 666, 777, 888, 999999 and 74?

22. If $4\frac{7}{9}$ yards of calico cost $5\frac{1}{3}$ shillings, what is the price of one yard?

23. How many acres of land in a field that is $73\frac{1}{4}$ rods long, and $49\frac{1}{3}$ rods wide?

24. If from a piece of broadcloth containing $\frac{4}{5}$ of $\frac{7}{8}$ of $32\frac{1}{3}$ yards, $\frac{1}{11}$ of $\frac{6}{7}$ of 49 yards be cut, how many yards, quarters, &c., will be left?

25. James Anthony bought of Oliver Edwards,

$126\frac{3}{4}$ pounds of rice, at $4\frac{1}{2}$ cents per pound.
$63\frac{1}{2}$ pounds of tea, at $66\frac{2}{3}$ cents per pound.
$95\frac{7}{8}$ gallons of molasses, at 29 cents per gallon.
$1\frac{23}{28}$ hundred weight of sugar, at $7\frac{1}{2}$ cents per pound.
$4\frac{7}{10}$ bushels of salt, at 64 cents per bushel.
$19\frac{2}{3}$ barrels of flour, at \$4.75 per barrel.
$96\frac{1}{4}$ pounds of coffee, at $9\frac{3}{4}$ cents per pound.

What was the amount of his bill?

26. If $\frac{2}{3}$ of $15\frac{3}{4}$ bushels of wheat cost $\$11\frac{1}{3}$, what is the price of 1 bushel?

27. Multiply in one line 6784.5 by 321.

28. Divide 8477905163 by 9999999; by 9999000.

29. A labourer received \$1614.00 for *3y.* $115\frac{1}{2}$*d.* wages. How much was that per day?

30. If $13\frac{7}{8}$ yards of broadcloth cost $\$62\frac{8}{9}$, what will be the cost of 1 yard? Of $7\frac{9}{10}$ yards?

31. How many arithmetics that are worth \$7.50 per dozen, can I buy for $\$10\frac{5}{8}$?

32. $74.\dot{6} \times 361.\dot{8} \div 261.5\dot{3}\dot{7} = ?$

33. Bought 11*C*. 3*C.ft.* of wood, for \$57.25. At what price must I sell it per cord, in order to gain \$9.75 on the whole?

34. Reduce to a common denominator $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{1}{5}$ of $\frac{4}{7}$, $\frac{2}{5}$ of $\frac{11}{21}$, $\frac{97}{315}$, and $\frac{76}{231}$.

35. What is the sum of $19\frac{4}{5}$, $2\frac{7}{8}$, $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of 8, $\frac{119}{120}$, and $63\frac{11}{15}$?

36. How much must I give for 49*T.* 5*cwt.* 1*qr.* of flour, at \$4.875 per barrel of 196 pounds?

37. A farmer sold $5\frac{3}{4}$ tons of hay at \$4.50 per bundle,

each bundle containing 5*cwt.* How many bundles were there, and what did he receive for the whole?

38. A., B. and C. traded in company. A. contributed \$19000, B. \$18500, and C. \$25750. What was the whole stock in trade, and what part of the gain should each man receive?

39. How many potatoes, at 25 cents a bushel, must be given in exchange for 163$\frac{3}{4}$ pounds of sugar, at 9$\frac{1}{2}$ cents a pound?

40. A. can do a piece of work in 8 days, and B. can do it in 12 days. What part can they both do in one day, and how long will it take them both to do the whole?

41. A merchant bought 17 bales of sheeting, each bale containing 18 pieces, and each piece 32$\frac{1}{4}$ yards. How much did he give for the whole, at 11$\frac{1}{2}$ cents a yard?

42. How much will it cost to carpet a floor 15*ft.* wide and 17*ft.* 6*in.* long,—the carpeting being a yard wide and worth \$1.50 per *yd.*?

43. The circumference of the earth's orbit is estimated at 567019740 miles. Then what distance does the earth move every second?

44. How many men can build a house in 1 day, that 25 men can build in 73 days?

45. A labourer's wages are \$1.75 per day. How many days must he work to pay for pasturing his cow 39$\frac{1}{2}$ weeks, at 37$\frac{1}{2}$ cents per week?

46. What is the diameter of the Earth, the circumference of any circle being 3.14159 times its diameter?

47. How many bottles, each holding 1$\frac{1}{2}$ pints, may be filled from a cask containing 127$\frac{1}{8}$ gallons of wine? How many drops are there in the cask?

48. How many times will a wheel that is 12$\frac{1}{2}$ feet in circumference, revolve in going from Boston to Providence, the distance being 39$\frac{1}{3}$ miles?

49. What must I give for 17 boxes of sugar, each weighing 2*cwt.* 1*qr.* 7*lb.*, at \$6.75 per cwt.?

50. What is the worth of a vessel, if $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{5}{8}$ of her cost \$875.00?

51. If a staff $9ft.$ $6in.$ long, cast a shadow of $12\frac{1}{3}$ feet, what is the height of a tree whose shadow at the same time measures $99\frac{4}{5}ft.$?

52. How much can a man save in a year, whose annual salary is $\$593\frac{1}{3}$, and whose daily expenses are $93\frac{3}{4}$ cents?

53. If $19\frac{2}{5}lb.$ of wool cost $\$5\frac{1}{6}$, what is the price of $23\frac{1}{3}lb.$?

54. How much must a man spend a day in order to use all his income, which is $1000 per annum?

55. Bought a house lot for $950.50. What is the value of $\frac{3}{4}$ of $\frac{1}{2}$ of $\frac{5}{9}$ of the lot?

56. How much cloth is there in a piece worth $\$127\frac{3}{8}$, at $\$4\frac{1}{6}$ per yd.?

57. A merchant bought 56 barrels of flour for $238, and paid for transportation $14.00; storage $4.75; drayage, and other expenses, $5.00. At what price must he sell it per barrel to gain $25 on the whole?

58. A cistern has 3 pipes, one of which would fill it in $4\frac{1}{3}$ hours, one in $3\frac{1}{2}$ hours, and the third would empty it in 5 hours. If they are all opened, in what time will the cistern be filled?

59. If $\frac{4}{5}$ of a ton of hay cost $\$11\frac{4}{15}$, what will $4\frac{5}{9}$ tons cost?

60. If a tax of $469 is levied on property valued at $140750, how much must be paid by a man whose property is worth $1900?

61. How many oxen at $\$95\frac{2}{3}$, horses at $120, and cows at $\$27\frac{1}{2}$, of each a like number, can be purchased for $4371.?

62. What is the value of $16\frac{1}{4}lb.$ of flour, at $4.75 per barrel of $196lb.$?

63. What will $9.763cwt.$ of rice cost, at $4\frac{1}{2}$ cents per pound?

64. How much must I give for $84\frac{1}{3}$ gallons of vinegar, at $6\frac{1}{4}$ cents a quart?

65. Bought $9\frac{1}{2}$ pieces of sheeting, each piece containing $30\frac{3}{4}$ yards, for $\$36\frac{33}{64}$. What was the price per yard?

66. If, when wheat is $1.00 per bushel, the $6ct.$ loaf

weighs $1\frac{3}{4}$*lb.*, what ought it to weigh when wheat is \$1.25 per bushel?

67. What would $96\frac{7}{8}$*lb.* of pork cost, at $7\frac{3}{4}$*cts.* per lb.?

68. What must be the length of a board that is $10\frac{1}{2}$ inches wide, to measure $9\frac{4}{7}$ square feet?

69. How long will it take 1 man to build a wall that 15 men can build in $4\frac{11}{18}$ days?

70. How many square feet in a board 2*ft.* 3*in.* wide, and 11*ft.* 7*in.* long?

71. The sum of two numbers is $96\frac{3}{17}$, and one of the numbers is $18\frac{5}{11}$; what is the other?

72. The greater of two numbers is $87\frac{1}{8}$, and their difference is 7.625. What is the less?

73. The less of two numbers is $14.7\dot{9}$, and their difference is $13\frac{5}{13}$. What is the greater?

74. The sum of two numbers is $169\frac{1}{2}$, and their difference is $28\frac{15}{16}$. Required the numbers. [Add half the difference to half the sum for the greater, and subtract half the difference from half the sum for the less number.]

75. What number must be taken from $\frac{1}{9}$ of $\frac{2}{16}$ of 8, to leave $\frac{3}{7}$ of $\frac{1}{2}$ of $\frac{1}{11}$ of $1\frac{3}{5}$?

76. A man left to his son $\$916\frac{1}{3}$ more than to his wife, and to his wife $\$159\frac{3}{4}$ more than to his daughter. The daughter's share was \$1299; what was the whole fortune?

77. If the divisor is 9327, the quotient $864\frac{1}{2}$, and the remainder $1591\frac{2}{3}$, what is the dividend?

78. What number must be added to $\frac{3}{19}$ of $62\frac{6}{7}$ to make $\frac{7}{11}$ of $\frac{3}{4}$ of $191\frac{13}{17}$?

79. If 1*cwt.* 2*qr.* 21*lb.* of indigo cost £33 17*s.* 7*d.*, how much will 6*cwt.* 3*qr.* 15*lb.* cost?

80. If 5.29 francs are worth \$1.00, what is the value of 688.31 francs?

81. Philadelphia, 4th mo. 29, 1844.

Simpson and Jones, Bought of M. Hart & Co.

18 Emerson's First Class Readers	at \$5.00 a dozen.
22 Colburn's Algebras	at \$8.00 a dozen.
9 gross steel pens	at \$2.25 a gross.
1200 slate pencils	at \$0.15 a hundred.

82. New York, Aug. 18, 1844.

Harcourt & Elliott, Bought of Wm. Thompson,

6 pieces of coarse cottons,	each $32\frac{1}{2}$*yds.*	at $10\frac{1}{2}$*cts.*
40 pieces " "	each $31\frac{3}{4}$*yds.*	at 11*cts.*
46 pieces fine "	each $28\frac{1}{4}$*yds.*	at $13\frac{7}{8}$*cts.*
49 pieces Merrimack prints,	each 30*yds.*	at $12\frac{1}{2}$*cts.*
28 pieces bleached muslins,	each $29\frac{5}{8}$*yds.*	at 15*cts.*
11 pieces Dover prints,	each $31\frac{1}{2}$*yds.*	at $13\frac{1}{2}$*cts.*

83. London, July 25, 1844.

Watson, Miller & Co., Bought of Joseph Merriam,

4 bags Surinam cotton,	each 3*cwt.* 1*qr.* 16*lb.*	at $21\frac{1}{4}$*d.*
3 bags " "	each 2*cwt.* 3*qr.* 27*lb.*	at $22\frac{1}{4}$*d.*
4 bags St. Domingo do.	each 3*cwt.* 0*qr.* 10*lb.*	at $14\frac{3}{4}$*d.*
4 bags Barbadoes do.	each 2*cwt.* 1*qr.* 15*lb.*	at $14\frac{1}{4}$*d.*
5 bags Pernambuco do.	each 2*cwt.* 2*qr.* 19*lb.*	at $21\frac{1}{2}$*d.*

84. Bristol, Sept. 5, 1844.

George Merriam & Brother, Bought of William Kent,

5*cwt.* 2*qr.* 9*lb.* hops,	at £4 12*s.* per *cwt.*
2*cwt.* 1*qr.* 9*lb.* "	at £4 19*s.* per *cwt.*
3*cwt.* 3*qr.* 27*lb.* "	at £5 2*s.* per *cwt.*
5*cwt.* 1*qr.* 18*lb.* "	at 1*s.* 3*d.* per *lb.*
7*cwt.* 3*qr.* 25*lb.* "	at 1*s.* $2\frac{3}{4}$*d.* per *lb.*

85. What is the yearly rent of $247\frac{1}{2}$ acres of land, at \$9.50 per acre?

86. A merchant bought $\frac{1}{2}$ of $\frac{5}{16}$ of a vessel, and afterwards sold $\frac{2}{3}$ of $\frac{7}{8}$ of his share for \$569. How much has he left, and what is its value at the same rate?

87. If $\frac{2}{5}$ of $\frac{5}{7}$ of $\frac{9}{16}$ of a farm be worth \$959.75, what is the whole farm worth?

88. What is the balance of the following account, the balance being the difference between the Debtor and Creditor amounts?

Dr.	*Cr.*
$1169.42\frac{3}{4}$	432.50
$875.18\frac{3}{4}$	$6911.47\frac{3}{8}$
9121.75	853.25
$638.12\frac{1}{2}$	$419.37\frac{1}{2}$
$991.14\frac{7}{8}$	$1640.61\frac{3}{4}$

89. If 939*cwt.* 3*qr.* 21*lb.* of sugar, sell for \$499.50, what is the price per *cwt.*? per *lb.*?

90. How much wool, at $37\frac{1}{2}$*cts.* a pound, will purchase 95 barrels of apples, at \$1.66$\frac{2}{3}$ per barrel?

91. What would be the value of $\frac{4}{13}$*cwt.* of gold, at \$13.75 per ounce Avoirdupois?

92. How many steps will a man take in $19\frac{1}{3}$ hours, if he walks $3\frac{1}{12}$ miles an hour, and each step measures $2\frac{3}{4}$*ft.*?

93. A certain room is $14\frac{1}{2}$*ft.* long, $11\frac{1}{3}$*ft.* wide, and $7\frac{5}{6}$*ft.* high. How many bushels will it hold, there being 2150.4 cubic inches in a bushel?

94. What is the value of $8131\frac{3}{4}$*lb.* of tea, at $62\frac{1}{2}$*cts.* per *lb*.

95. What is the price of 1*oz.* of indigo, if $4\frac{5}{9}$*lb.* cost \$$13\frac{2}{3}$?

96. A rectangular farm contains 10*A.* 3*R.*, and it measures in front 39 rods. What is the length of one of the sides?

97. A merchant gained \$1920 a year, for the first three years after commencing business; for the next four years he gained \$2370.50, and for the next two he lost \$1875.75 a year. How much had he gained at the expiration of the ninth year?

98. How many cents weigh 1*cwt.* 1*qr.* 15*lb.* 15*oz.*, if the weight of one cent is $\frac{4}{135}$*lb.*?

99. $5914\frac{1}{2}$ is $\frac{2}{3}$ of $\frac{7}{11}$ of $\frac{1}{4}$ of what number?

100. Add $\frac{1}{7}$ of $\frac{3}{5}$ of $\frac{9}{11}$ of $16\frac{1}{2}$ years, $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{6}{11}$ of 4 days; $2\frac{17}{18}$ minutes, and $\frac{1}{3}$ of 62.49 seconds.

101. Nine men bought 27 barrels of flour, at \$$5\frac{3}{7}$ per barrel. How much did each pay?

102. A merchant who owned $\frac{1}{3}$ of $\frac{7}{8}$ of a ship, gave $\frac{2}{5}$ of his share to his son, and sold $\frac{1}{4}$ of the remainder for 1375. What was the value of the whole vessel, at the same rate?

103. If $11\frac{3}{7}$ gallons of wine cost $12\frac{1}{5}$ dollars, what will $2\frac{1}{8}$ gills cost?

104. A silversmith has 4 tea-pots, each weighing 1*lb.* 8*oz.* 12*dwt.* 6*gr.*; $2\frac{1}{3}$ dozen silver spoons, each weighing 2*oz.* 19*dwt.* 18*gr.*; and 37 tea-spoons, each weighing 16*dwt.* 7*gr.* What is the weight of the whole, and what is their value at \$19.00 per lb.?

105. How many gills in 16 casks, each holding $35\frac{1}{7}$ gallons?

106. If 3*cwt.* 3*qr.* 3*lb.* of pork cost \$19.50, what will $19\frac{3}{4}$*lbs.* cost?

107. A boy being asked how many marbles he had, replied, "If I had as many more, and $\frac{1}{2}$ as many more, I should have 45." How many had he?

108. What is the cost of a load of wood that is 9*ft.* long, 4*ft.* wide, and $4\frac{1}{2}$*ft.* high, at $6.62\frac{1}{2}$ per cord?

109. A boy having some peaches, found, after giving away 2 more than $\frac{1}{3}$, that 1 more than $\frac{1}{2}$ remained. How many had he at first?

110. Add $16\frac{5}{9}$, $4\frac{2}{3}$, $17\frac{1}{6}$, $491\frac{3}{4}$, and $21\frac{7}{16}$.

111. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ and $\frac{1}{7}$ of a certain number, is equal to $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{1}{2}$ of $\frac{4}{7}$ of $16858\frac{1}{5}$. What is the number?

112. Add 5.793*cwt.*, $4\frac{7}{16}$*cwt.*, 3*cwt.* 3*qr.* 15*lb.* and 1*qr.* 9*lb.*, and reduce the result to ounces.

113. $916.4 \times .0015 \div 363.18 = ?$

114. $.0091 \times .00037 \div 1950000 = ?$

115. Reduce to their lowest terms $\frac{646}{238}$; $\frac{112}{532}$; $\frac{232}{843}$; $\frac{132}{627}$; $\frac{188}{628}$.

116. What is the value of a farm 189.5 rods long, and 150 rods wide, at \$37.50 per acre?

117. How many times will a half-seconds' pendulum tick in $\frac{2}{3}\frac{1}{7}$ of a year?

118. What is the value of £63.795 at $\frac{1}{2}$ cent per farthing?

119. What part of a solar year is 11 days 5 hours?

120. What part of a rod is 1*ft.* 7.9*in.*?

121. What part of a yard is 3*qr.* 1*na.* 1.5*in.*?

122. What part of £4 11*s.* 3*d.* is £1 5*s.* $2\frac{3}{4}$*d.*?

123. What part of 5℔ 7℥ 3ʒ is 2℔ 4℥ 1℈?

124. What part of 4*gal.* 3*qt.* is 2*qt.* 1*pt.* 3*gi.*?

125. What part of 4*T.* 3*cwt.* 1*qr.* is $2\frac{3}{7}$*T.*?

126. What part of $2\frac{5}{9}$*cwt.* is $18\frac{3}{5}$*lb.*?

127. At \$1.75 per square yard, how much must I give for carpeting a floor 15 3′ long, and 12*ft.* wide?

128. What will $\frac{2}{3}$ of $15\frac{3}{8}$ yards of broadcloth cost, at $\frac{3}{4}$ of $7.25 per yard?

129. If A. can do a piece of work in 6 days, B. can do it in 9 days, and C. can do it in 12 days, in what time can it be done if they all work together?

130. What number is that, which, increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, makes 48?

131. In a certain orchard $\frac{1}{2}$ the trees bear apples, $\frac{1}{4}$ bear peaches, $\frac{1}{6}$ bear cherries, and the remaining 30 bear pears. How many trees are there in the orchard, and how many of each sort?

132. What is the value of $\frac{5}{11}$ of a ton?

133. How many inches in $\frac{5}{18}$ of a mile?

134. Reduce 11*oz.* 7*dwt.* 3*gr.* to the fraction of a ℔.

135. Reduce 4.79*oz.* to the fraction of a pound Avoirdupois.

136. Reduce 38.756*lb.* to the fraction of a ton.

137. Reduce 4.763*in.* to the fraction of a mile.

138. Reduce $\frac{3}{5}$ of a *qr.* to the decimal of a ton.

139. Reduce $4\frac{7}{8}$ pence to the decimal of a pound.

140. Reduce 3.725 shillings to the fraction of a pound.

141. Reduce 59.63 minutes to the fraction of a day.

142. Reduce $\frac{5}{9}$ of a day to hours, minutes, &c.

143. If $2\frac{5}{7}$ pounds of sugar cost $11\frac{8}{9}$ cents, what will $5\frac{1}{4}$ pounds cost?

144. If 9 men can reap a field in $3\frac{1}{2}$ days, how long will it take two men to reap it?

145. How much rice, at $4\frac{3}{4}$ cents a pound, can be bought for $1.50?

146. If $29\frac{1}{3}$ gallons run from a cistern in an hour, how many pints will run out in $16\frac{5}{8}$ hours?

147. What is the price of $3\frac{3}{8}$*cwt.* of rice, at $4\frac{1}{2}$*cts.* per pound?

148. At $12\frac{1}{2}$ cents a yard, what will be the cost of 13*y.* 3*qr.* 3*na.* of sheeting?

149. If $\frac{7}{9}$ of a bushel of wheat cost $0.75, what will be the price of $3\frac{3}{4}$ pecks?

150. How far will a pigeon fly in 3*h.* 45*m.* 45*sec.*, at the rate of 25 miles an hour?

151. How many cents would reach from Philadelphia to Washington, supposing the distance to be 132½ miles, and the diameter of a cent 1.15 inches?

PROOF BY CASTING OUT 9's.

The figure 9 has the curious property of exactly dividing any number, when the sum of its digits is divisible by 9, and on this property is founded the mode of proof known as *casting out the nines.* This is done by adding the figures which compose any number, and rejecting 9 from the sum as often as possible. Thus, if we wish to cast out the 9's from 7683217, we say 7 and 6 are 13 *less* 9 *are* 4 and 8 are 12 *less* 9 *are* 3 and 3 are 6 and 2 are 8 and 1 are 9 *less* 9 *are* 0 and 7 are 7. We then know that if 7683217 is divided by 9, there will be a remainder, 7:

To prove any operation, we must reject the nines from each of the original numbers, perform the operation with the remainder, and reject the nines from the result, and also from the original result. If the work is right, the final remainders will be equal. The following examples will show the application of the rule:

Addition.	Rem.
168412	4
49037	5
684673	7
902122	16
Final rem. 7 =	7

Subtraction.	
23849047	1+9
9108736	7
14740311	3
Final rem. 3=3	

Multiplication.	Rem.
83869	7
233	8
251607	56
251607	
167738	
19541477	
Final rem. 2 =	2

Division.		
	268) 3462 (12	
	268	
	782	
	536	
	246	Rem.
Divd.—Rem.=	3216	3
Div. Rem. × Quot. Rem.=	21	3

In the example of subtraction, the remainder of the minuend being less than that of the subtrahend, we increase the upper number by one of the rejected nines.

The proof is not infallible, as, if any error is 9 or some multiple of 9, it will not be detected. There is, however, great advantage in the facility thus afforded for discovering errors arising from transposition.

TRANSPOSITION.

The difference between any number, and the same number transposed, is divisible by 9. Thus, 723—327, 1680—0681, 231—123, are each exactly divisible by 9. Therefore, if we find in comparing the books of a counting-room or banking-house, that they do not agree, and the amount of their disagreement is divisible by 9, we know that it *may* have arisen from a transposition. We shall thus frequently be enabled to discover an error readily, which would otherwise have required a long and tedious examination.

CHAPTER VIII.

PRACTICE.

In PRACTICE, many questions arise that can be solved more readily than by adopting either of the foregoing rules. Most of the operations of business, in which compound numbers are concerned, may be abbreviated by first finding values for the highest denomination, and considering the lower denominations as aliquot parts of the higher.

TABLES OF ALIQUOT PARTS.

Of a dol.		*Of a £.*			*Of a shi.*		*Of a ton.*			*Of a cwt.*			*Of a year.*		
cts.	$	*s.*	*d.*	£	*d.*	*s.*	*cwt.*	*qr.*	*ton.*	*qr.*	*lb.*	*cwt.*	*mo.*	*d.*	*y.*
50	$=\frac{1}{2}$	10		$=\frac{1}{2}$	6	$=\frac{1}{2}$	10		$=\frac{1}{2}$	2		$=\frac{1}{2}$	6		$=\frac{1}{2}$
$33\frac{1}{3}$	$=\frac{1}{3}$	6	8	$=\frac{1}{3}$	4	$=\frac{1}{3}$	5		$=\frac{1}{4}$	1		$=\frac{1}{4}$	4		$=\frac{1}{3}$
25	$=\frac{1}{4}$	5		$=\frac{1}{4}$	3	$=\frac{1}{4}$	4		$=\frac{1}{5}$		16	$=\frac{1}{7}$	3		$=\frac{1}{4}$
20	$=\frac{1}{5}$	4		$=\frac{1}{5}$	2	$=\frac{1}{6}$	2	2	$=\frac{1}{8}$		14	$=\frac{1}{8}$	2	12	$=\frac{1}{5}$
$16\frac{2}{3}$	$=\frac{1}{6}$	3	4	$=\frac{1}{6}$	$1\frac{1}{2}$	$=\frac{1}{8}$	2		$=\frac{1}{10}$		8	$=\frac{1}{14}$	2		$=\frac{1}{6}$
$12\frac{1}{2}$	$=\frac{1}{8}$	2	6	$=\frac{1}{8}$	1	$=\frac{1}{12}$	1	1	$=\frac{1}{16}$		7	$=\frac{1}{16}$	1	15	$=\frac{1}{8}$
10	$=\frac{1}{10}$	2		$=\frac{1}{10}$	$\frac{3}{4}$	$=\frac{1}{16}$	1		$=\frac{1}{20}$		4	$=\frac{1}{28}$	1	10	$=\frac{1}{9}$
$8\frac{1}{3}$	$=\frac{1}{12}$	1		$=\frac{1}{20}$	$\frac{1}{2}$	$=\frac{1}{24}$		2	$=\frac{1}{40}$		2	$=\frac{1}{56}$	1	6	$=\frac{1}{10}$
$6\frac{1}{4}$	$=\frac{1}{16}$		10	$=\frac{1}{24}$	$\frac{1}{4}$	$=\frac{1}{48}$	8		$=\frac{2}{5}$	1	4	$=\frac{2}{7}$	4	24	$=\frac{2}{5}$
5	$=\frac{1}{20}$		8	$=\frac{1}{30}$	9	$=\frac{3}{4}$	12		$=\frac{3}{5}$	1	20	$=\frac{3}{7}$	7	6	$=\frac{3}{5}$
4	$=\frac{1}{25}$		4	$=\frac{1}{60}$	8	$=\frac{2}{3}$	16		$=\frac{4}{5}$	2	8	$=\frac{4}{7}$	9	18	$=\frac{4}{5}$
$3\frac{1}{3}$	$=\frac{1}{30}$		3	$=\frac{1}{80}$	$7\frac{1}{2}$	$=\frac{5}{8}$	15		$=\frac{3}{4}$	2	24	$=\frac{5}{7}$	4	15	$=\frac{3}{8}$
2	$=\frac{1}{50}$		2	$=\frac{1}{120}$	$4\frac{1}{2}$	$=\frac{3}{8}$	7	2	$=\frac{3}{8}$	3	12	$=\frac{6}{7}$	7	15	$=\frac{5}{8}$

Similar tables may be made to any required extent, but these are sufficient to show their application.

RULE.

Assume the price at some unit higher than the given price, and take aliquot parts of the assumed price for the answer.

TO FIND THE VALUE OF A QUANTITY OF SEVERAL DENOMINATIONS.—Multiply the price by the integers of the highest denomination, and take aliquot parts for the lower denominations.

EXAMPLES FOR THE BOARD.

What is the value of 11*cwt.* 3*qr.* 17*lb.* of sugar, at £1 3*s.* 6*d.* *per cwt.*?

	£		*s.*		*d.*	
2*qr.*=½*cwt.*	1	"	3	"	6	price of 1*cwt.*
16*lb.*=$\frac{1}{7}$*cwt.*					11	
	12	"	18	"	6	price of 11*cwt.*
1*qr.*=½2*qr.*			10	"	9	price of 2*qr.*
			5	"	$10\frac{1}{2}$	price of 1*qr.*
1*lb.*=$\frac{1}{16}$16*lb.*			3	"	$4\frac{2}{7}$	price of 16*lb.*
					$2\frac{15}{56}$	price of 1*lb.*
	12	"	39	"	$6\frac{3}{56}$	price of 11*cwt.* 3*qr.* 17*lb.*

What is the interest of \$187.50 for 3*yr.* 9*mo.* 19*d.* at .07 per year?

		187.50	
		.07	
6*mo.*	½	13.1250	int. for 1*yr.*
		3	
		39.375	int. for 3*yr.*
		$6.562\frac{1}{2}$	int. for 6*mo.*
3*mo.*	½	$3.281\frac{1}{4}$	int. for 3*mo.*
15*d.*	$\frac{1}{6}$	$.546\frac{7}{8}$	int. for 15*d.*
3*d.*	$\frac{1}{5}$	$.109\frac{3}{8}$	int. for 3*d.*
1*d.*	$\frac{1}{3}$	$.036\frac{11}{24}$	int. for 1*d.*
		$49.911\frac{11}{24}$	int. for 3*yr.* 9*mo.* 19*d.*

What is the value of 96*lb*. of tea, at 3*s*. 10$\frac{3}{4}$*d*. per *lb*.

The price at £1 per *lb*. would be		£96
at	3*s*. 4*d*.= $\frac{1}{6}$ £	16
at	4*d*.= $\frac{1}{10}$ 3*s*. 4*d*.	1 " 12
at	2*d*.= $\frac{1}{2}$ 4*d*.	16
at	$\frac{1}{2}$*d*.= $\frac{1}{4}$ 2*d*.	4
at	$\frac{1}{4}$*d*.= $\frac{1}{2}$ of $\frac{1}{2}$*d*.	2
at 3*s*. 10$\frac{3}{4}$*d*.		£19 " 4*s*.

1. What is the price of 89$\frac{3}{4}$*yds*. of broadcloth, at \$4.75 per *yd*.?

2. What is the price of 127$\frac{5}{8}$ pounds of sugar, at 12$\frac{1}{2}$*cts*. per *lb*.?

3. What is the value of 49*A*. 3*R*. 15*r*. of land, at \$125 per acre?

4. What is the value of 15*T*. 13*cwt*. of hay, at 87$\frac{1}{2}$*cts*. per *cwt*.?

5. What is the value of 96*yds*. 3*qr*. 3*na*. of broadcloth, at £1 2*s*. 6*d*. per yard?

6. What will 17*T*. 11*cwt*. 2*qr*. 21*lb*. of iron cost, at \$19.75 per ton?

7. What will 7*cwt*. 2*qr*. 11*lb*. of sugar cost, at \$7.62$\frac{1}{2}$ per *cwt*.?

8. What is the cost of 389*bu*. 2*pk*. 4*qt*. of wheat, at \$1.12$\frac{1}{2}$ per bushel?

9. What is the cost of 163*A*. 2*R*. 25*r*. of land, at \$15.75 per acre?

10. If 1 *lb*. of beef costs 6$\frac{1}{4}$ cents, what will 16*cwt*. 2*qr*. 7*lb*. cost, at the same rate?

11. If an ounce of indigo costs 37$\frac{1}{2}$ cents, what must I give for 1*cwt*. 1*qr*. 14*lb*. 6*oz*., at the same rate?

12. What is the value of 364*yd*. 3*qr*. 1*na*. of sheeting, at 12$\frac{1}{2}$*cts*. a yard?

13. What is the value of 379$\frac{1}{2}$*lb*. of coffee, at 10*cts*. per pound? At 12$\frac{1}{2}$*cts*.?

14. Bought 76*bu*. 3*pk*. of potatoes, at 37$\frac{1}{2}$*cts*. a bushel;

19*bu.* 2*pk.* of wheat, at $1.10 a bushel; 37*bu.* 1*pk.* of barley, at 62½*cts.* a bushel; and 10*T.* 15*cwt.* of hay, at $16.00 a ton. What was the amount of the whole?

15. What is the value of 11*cwt.* 3*qr.* 9*lb.* of sugar, at £2 3*s.* 6¾*d.* per hundred weight?

16. What is the price of 36*lb.* 5*oz.* of tea, at 87½*cts.* per pound?

17. What is the value of 191*bu.* 3½*pk.* of apples, at $1.25 per bushel?

18. What is the value of 187*yd.* 1*qr.* 2*na.* of silk, at 93¾ cents a yard?

19. What is the interest of $174.50 for 4*yr.* 6*mo.* 27*d.*, at .05 per year?

20. What is the interest of $2375 for 5*yr.* 11*mo.* 23*d.*, at .055 per year?

21. What is the interest of $4814.25 for 3*yr.* 7*mo.* 14*d.*, at .06 per year? At .075 per year?

There are a variety of other contractions that may frequently be adopted in practice. A few are given below, which will often be found useful.

(1.) When the multiplier consists of any number of 9's, increase it by 1, and subtract the multiplicand from the product. Thus, $18473\times9999=184730000-18473=184728 1527$.

(2.) To multiply by 5, divide the multiplicand by .2. Thus, $187\times5=187\div.2=935$. To divide by 5, multiply the dividend by .2.

(3.) To multiply by 25, divide the multiplicand by .04. Thus, $1289\times25=1289\div.04=32225$. To divide by 25, multiply the dividend by .04.

(4.) To multiply by 75, multiply by 100 and subtract $\frac{1}{4}$ of the product. Thus, $18645\times75=1864500-466125=1398375$. To divide by 75, divide by 100, and add $\frac{1}{3}$ of the quotient.

(5.) To multiply by 125, divide the multiplicand by .008. Thus, $1641\times125=1641\div.008=205125$. To divide by 125, multiply the dividend by .008.

(6.) To multiply by 375, divide by .008, and multiply the quotient by 3. Thus, $294\times375=294\div.008\times3=110250$. To divide by 375, multiply by .008 and divide by 3.

(7.) To multiply by 625, divide the multiplicand by .0016. Thus, $4812\times625=4812\div.0016.=300750$. To divide by 625, multiply the dividend by .0016.

(8.) To multiply by 875, multiply by 1000 and subtract $\frac{1}{8}$ of

the product. Thus, $7539 \times 875 = 735000 - 91875 = 643125$. To divide by 875, divide by 1000 and add $\frac{1}{7}$ of the quotient.

(9.) To multiply by any number within 12 of 100, 1000, &c., annex to the multiplicand as many zeroes as there are figures in the multiplier, and subtract as many times the multiplicand as are equivalent to the excess of 100, 1000, &c., over the multiplicand. Thus, $24796 \times 99989 = 2479600000 - (11 \times 24796) = 2479327244$.

(10.) To square* a number ending in 5, multiply the number of tens by one more than itself, and place 25 at the right of the product. Thus, $3 \times 4 = 12$, and $35 \times 35 = 1225$; $12 \times 13 = 156$, and $125 \times 125 = 15625$; $6 \times 7 = 42$, and $65 \times 65 = 4225$.

(11.) When the tens in two numbers are alike, and the sum of the units is 10, to obtain the product multiply the number of tens by one more than itself for the hundreds, and place the product of the units at the right of this product, for the tens and units. Thus, $4 \times 5 = 20$, and $43 \times 47 = 2021$; $42 \times 48 = 2016$; $44 \times 46 = 2024$; $7 \times 8 = 56$, and $72 \times 78 = 5616$; $71 \times 79 = 5609$, &c.

(12.) The sum of two numbers multiplied by their difference, is equal to the difference of their squares. Hence we may readily find the product of two numbers, one of which is as much above as the other is below, a certain number of tens. Thus, $87 \times 73 = (80+7) \times (80-7) = 80^2 - 7^2 = 6400 - 49 = 6351$.

(13.) To square any number between 50 and 60, add the units of the given number to 25 for the hundreds, and annex the square of the units for the tens and units. Thus, for the square of 51; $25+1=26$ *hundreds*, and $1 \times 1 = 1$; hence $51 \times 51 = 2601$. In like manner $53 \times 53 = 2809$; $59 \times 59 = 3481$.

(14.) When one figure of the multiplier is an aliquot part of one or more of the remaining figures, the work may be abbreviated as in the following example:

Multiply 489.137 by 7261.8.

```
     489.137
     7261.8
    --------
   2934822     =prod. by 6.
     8804466   =prod. by 3×6=prod. by 18.
  35217864     =prod. by 4×18=prod. by 72.
  ------------
  3552015.0666
```

We see at once that 18 is a multiple of 6, and 72 is a multiple of 18. Therefore, multiplying first by 6, we take 3 times the product for the product by 18, and 4 times the product by 18, for the product by 72.

---

* The product of any number multiplied by itself, is called the square of the number.

(15.) In the ordinary mode of determining the greatest common divisor of two numbers, any prime factor or square number, contained in one number but not in the other, or any prime factor or square number in a remainder that is not in the preceding divisor, may be rejected, and the work thus abbreviated. For example, let the greatest common measure of 689 and 901 be required.

```
            689 ) 2279 (3
                  2067
                  ----
                4 ) 212
                  ----
G. C. Meas.   53 ) 689 (13
                   53
                   ---
                   159
                   159
                   ---
```

Here the square number 4 is a factor of 212, and not of 689. We therefore divide 212 by 4, and immediately obtain the greatest common measure. In the application of this principle to the reduction of fractions, we observe that 53 divides 689, 13 times, and it divides 212, 4 times. It therefore divides $3 \times 689 + 212$ or 2279, $3 \times 13 + 4$ or 43 times.

Therefore $\frac{689}{2279} = \frac{13}{43}$.

Reduce $\frac{457}{563}$ to its lowest terms.

```
457 ) 563 ( 1
      457
      ---
      106 = 2 × 53
```

Neither 2 nor 53 being factors of 457, the fraction is already in its lowest terms.

(16.) We have already seen the application of cancelling, in the reduction of fractions to their lowest terms. The principle is applicable in all cases in which the product of one set of numbers is to be divided by the product of another set. If either multiplier contains a factor of either divisor, the factor may be removed from both without altering the result. Upon this truth is founded a ready mode of solving many intricate questions, which will be explained more fully in the chapter on Proportion.

22. Multiply 576.3 by 99; by 999000.

23. Multiply 7.894 by 5; by 25; by 7500.

24. Multiply 48.302 by 1250; by 375000.

25. Divide 1879.4 by 5; by 250; by 75.

26. Divide 4449.17 by 125; by 375.

27. Multiply 3.0872 by 525125, by adding three partial products.

28. Multiply 41909 by 999625125, in the most expeditious manner.

29. Multiply 89443 by 625; by 875.

30. Divide 141.982 by 625; by 875.

31. Multiply 89443 by 625875.

32. Multiply 283172 by 9992; by 991.

33. What is the square of 15? of 85? of 115?

34. What is the product of 73×77? 12×18? 44×46?

35. What is the product of 81×89? 75×75? 34×36?

36. What is the product of 16×24? 19×21? 35×45? 89×71? 67×53? 78×82? 96×84? 113×107? 112×128?

37. What is the square of 58? of 56? 52? 55? 57? 54?

38. Find the greatest common divisor of 804 and 938; of 741 and 1083; of 1343 and 1817.

39. Reduce each of the following fractions to its lowest terms: $\frac{301}{473}$, $\frac{781}{1207}$, $\frac{667}{899}$, $\frac{1147}{1333}$, $\frac{941}{1711}$.

40. Multiply 476384 by 9995125625.

CHAPTER IX.

PERCENTAGE.

It is customary to estimate all sums paid as commission or brokerage for the sale or purchase of property, insurance against loss by fire or otherwise, interest for the use of money, &c., at a given number of hundredths, called the *rate per cent.* (from the Latin *per centum*, signifying *by the hundred*). Thus, 6 per cent. is .06; $5\frac{1}{4}$ per cent. is $.05\frac{1}{4}$ or .0525; $7\frac{1}{3}$ per cent. is $.07\frac{1}{3}$ or $.07\dot{3}$. To compute any percentage, *Multiply by the rate expressed decimally.*

Commission is an allowance of a certain percentage to a Factor, Correspondent, or Broker, for buying or selling property.

1. A broker sold goods at a commission of $4\frac{1}{2}$ per cent., for which he received \$1963.50. What was the amount of his commission, and how much did he pay to his employer?

2. How much shall I receive for my real estate, by sell-

ing it for \$9550.00, after allowing a commission of $3\frac{1}{3}$ per cent.?

3. My correspondent has expended \$4762.00 in the purchase of cloths, by my order. What will he receive for the transaction, if I allow him $2\frac{3}{4}$ per cent. on the whole amount?

4. What is the commission on £963 17*s.* 6*d.*, at $4\frac{2}{3}$ per cent. [Multiply by $\frac{14}{3}$ and divide by 100.]

Insurance is a security against loss by fire or otherwise, and is generally obtained by the payment of a certain percentage, called the **Premium.**

The instrument by which insurance is effected, is called the **Policy.**

5. A farmer insured his house for \$950.00 at $1\frac{1}{4}$ per cent., his barn at \$375.00 at $1\frac{1}{2}$ per cent., and his granary and sheds for \$275.00 at 1 per cent. What premium did he pay for the whole?

6. An English merchant effected an insurance of £5163 10*s.* on the ship Mary Ann, at a premium of $1\frac{3}{4}$ per cent. What did he pay, including a charge of 3*s.* 6*d.* for the policy?

7. What premium must be paid for insuring \$9795.00 on a warehouse, at $2\frac{1}{5}$ per cent.?

8. What amount of premium must I pay to insure the following property for one year: \$10875.00 on my store, and \$4500.00 on the merchandise in the same, at $\frac{3}{4}$ per cent.; \$4650.00 on my house, and \$1750.00 on the furniture, books and clothing contained in it, at $\frac{1}{2}$ per cent.; and \$975.00 on my barn, at $\frac{5}{8}$ per cent.?

Taxes are certain amounts levied on all citizens for the support of government. The poll tax is assessed on every male citizen above a certain age specified by law, and its amount is fixed by statute. The property tax is estimated at a certain percentage of its assessed value.

9. When taxes are rated at $1\frac{1}{4}$ per cent. upon all real and personal estate, what amount will be paid by a man whose property is valued at \$5680.00, and who pays for 3 polls at \$0.75?

10. The poll tax being \$1.50, and the property tax

\$4.50 on every \$100.00, how much must I pay for \$4675.00 real estate, \$963.00 personal property, and 2 polls?

11. At $3\frac{3}{4}$ per cent. what amount of tax must be paid by property valued at \$18875.25? How much would be paid on the same property, at $4\frac{1}{3}$ per cent.? At $\frac{2}{3}$ per cent.? At $\frac{3}{7}$ per cent.?

STOCKS are sums of money invested in government funds or corporations, and are bought and sold by ***shares.*** The ***par value*** of a share is the original cost. When a share will sell for more than the original cost, it is said to be ***above par***, or ***at an advance;*** when for less, ***below par***, or ***at a discount.***

12. What must I pay for 27 shares of the Boston and Worcester Rail Road, the stock being $16\frac{1}{2}$ per cent. above par, and the par value \$100 per share?

13. The par value of United States Bank stock, was \$100 per share. How much was received for 31 shares, at $9\frac{1}{4}$ per cent. below par?

14. A man purchased \$5000 Pennsylvania stock at par, and sold it at a discount of $19\frac{1}{2}$ per cent. How much did he receive for the whole?

15. What must I pay for 43 shares of bank stock, at an advance of $7\frac{5}{8}$ per cent. on the par value, which is \$250 per share?

16. What is the value of \$9500 of stock, at 25 per cent. advance? At $19\frac{1}{3}$ per cent. discount? At 50 per cent. below par?

GAIN AND LOSS are frequently estimated by percentage.

17. How much per cent. do I gain, by selling at 90*cts.* a pound, tea that cost me 75 cents? [The gain is 15*cts.*, or $\frac{15}{75}$ of the prime cost. This fraction reduced to a decimal, gives .2, or .20.] ***Ans.*** 20 ***per cent.***

18. 3.5 is what per cent. of 31? ***Ans.*** $19\frac{9}{31}$ ***per. cent.***

19. What per cent. of 63 is 7? 12? 13.7? 49.25?

20. Bought 18.75 yards of broadcloth, at \$4.00 per yard, and sold the whole for \$83.50. What did I gain, and how much per cent.?

21. A grocer sold sugar at $7\frac{1}{2}$ cents a pound, which

was 20 per cent. more than it cost him. What was the prime cost? In other words, 0.075 is 1.20 times what number?

22. 2.975 is .85 of what number? Then, if I lose 15 per cent. by selling broadcloth at $2.975 per yard, what was the prime cost?

23. A draper sold cloth at $12\frac{1}{2}$ per cent. less than it cost him, thereby losing $62\frac{1}{2}$ cents a yard. What did he pay for the cloth?

24. When sugar is $6.50 per *cwt.*, what must be the price per *lb.* to gain 40 per cent.?

25. A merchant purchased 96 bushels of wheat, at $0.95 per bushel, and paid $1.25 for carting it from the farm. For how much must he sell one half of it to gain 18 per cent.?

26. Bought in Jamaica 17 hogsheads of molasses, each holding 118*gal.* 2*qt.* at 20 cents a gallon; paid for freight $1.25 per hogshead; for insurance $8.06; and for duties $35.154. At what price per gallon must I sell it to gain 25 per cent.?

27. Bought 13*cwt.* 3*qr.* 17*lb.* of coffee, at $9\frac{1}{2}$ cents a pound, but it having been damaged, I sold the whole for $125.00. How much did I lose, and what per cent.?

28. Bought $39\frac{1}{2}$ reams of paper for $$162.93\frac{3}{4}$. For how much must I sell it per ream, to gain $33\frac{1}{3}$ per cent.?

Duties are imposts levied by government on goods imported into a country. Before computing them, certain deductions are made. For leakage and breakage, 2 per cent. is allowed on the gauge on all merchandise paying duty by the gallon, contained in casks; 10 per cent. on all beer, ale and porter, in bottles, and 5 per cent. on all other liquors in bottles.

Draft is an allowance made for waste. The following is the present legal rate:

	On 1*cwt.* or 112*lb.*			1*lb.*
Above	1 —	and under	2*cwt.*	2*lb.*
"	2 —	"	3 —	3*lb.*
"	3 —	"	10 —	4*lb.*
"	10 —	"	18 —	7*lb.*
	18 —	and upwards,		9*lb.*

TARE is an allowance made for the weight of the box, cask, &c., containing the merchandise, and is made after deducting the draft. The weight before making the deductions, is called the *gross weight*,—after the deductions are made, the *net weight*. Duties are computed on the net weight.

29. What is the duty on 5*cwt.* 3*qr.* 11*lb.* of currants, tare 12 per cent., duty 3 cents a pound?

30. What is the duty on 156*lb.* of chocolate, tare 10 per cent., duty 4 cents per pound?

31. What is the duty on 459*gal.* 2*qt.* of wine, at 35*cts.* a gallon?

32. What is the duty on $2\frac{2}{5}$*cwt.* of indigo, tare 12 per cent., duty 5*cts.* a pound?

33. What is the duty on $3\frac{5}{8}$*cwt.* of pepper, tare 5 per cent., duty 5*cts.* a pound?

INTEREST.

INTEREST is an allowance of a certain per cent. for the use of money. The *Principal* is the sum on which the interest is paid. The *Rate* is the percentage paid annually. It is regulated by statute, the usual rate being 6 per cent.* The *Amount* is the sum of principal and interest.

When interest is charged only on the original principal, it is called SIMPLE INTEREST. But when it is also charged on the interest, as it becomes due, it is called COMPOUND INTEREST.

SIMPLE INTEREST.

The usual rate is .06, and this rate is always to be understood, unless some other is expressed.

* In each of the New England States, in New Jersey, Pennsylvania, Delaware, Maryland, Virginia, North Carolina, Tennessee, Kentucky, Ohio, Indiana, Illinois, Missouri, Arkansas, and the Dist. of Columbia, and on U. S. notes, the rate is 6 per cent. In N. York, S. Carolina, Michigan, Wisconsin and Iowa, it is 7 per cent. In Georgia, Alabama, Mississippi and Florida, 8 per cent. In Louisiana, 5 per cent., though the bank interest is .06, and conventional interest may be as high as .10. In Maryland the interest on tobacco contracts is .08. In Mississippi, Missouri and Arkansas, the interest by agreement may be as high as .10, and in Illinois, Wisconsin and Iowa, as high as .12.

If the rate is .06 for one year, for one month it will be $\frac{1}{12}$ as much, or .005; for 6 days, or $\frac{1}{5}$ of 1 month, it will be $\frac{1}{5}$ as much as for one month, or .001. For 1 day, it will be $\frac{1}{6}$ as much as for 6 days, or $\frac{1}{6}$ of .001, which is $\frac{1}{6000}$. Hence the

BANK RULE.

Multiply the principal by the number of days, and divide by 6000; the quotient will be the interest at 6 per cent.

For any other rate than 6 per cent., compute the interest in the same manner, and add or subtract such part as may be required. For 4 per cent. subtract $\frac{1}{3}$; for $4\frac{1}{2}$ per cent. subtract $\frac{1}{4}$; for 5 per cent. subtract $\frac{1}{6}$; for 7 per cent. add $\frac{1}{6}$, and so on.

This rule is the one generally employed by banks and merchants. It estimates the year at 12 months of 30 days, and consequently gives the interest a trifle too large. If it is desired to obtain the exact interest, *deduct* $\frac{1}{73}$ from the sum found by the rule.

DECIMAL RULE.

Multiply the time, expressed in years and decimals of a year, by the annual rate, and multiply the principal by this product.

EXAMPLES FOR THE BOARD.

Find the bank interest of $1963.50 from Jan. 29th, to May 2d.

Find the exact interest of $4725.00 for 5*mo.* 19*d.*, by each of the above rules.

1. What is the bank interest on $950, for 6 months, at 6 per cent.?

2. What is the decimal interest on $1375, for 11*mo.* 5*d.*, at 5 per cent.?

3. What is the exact interest on $2000 for 475 days, at 4 per cent.?

4. What is the exact interest on $1500, from Aug. 18th, 1843, to Sept. 7th, 1844, at 6 per cent.?

5. What is the bank interest on $942.50, from Feb. 11th, 1844, to Dec. 3d, 1844, at 6 per cent.?

6. What is the bank interest on $236.35, for 63 days, at $4\frac{1}{2}$ per cent?

7. What is the bank interest on $75.10 for 93 days, at 7 per cent.?

8. What is the decimal interest on $4000, from June 1st, 1841, to May 13th, 1844, at 6 per cent.?

9. What is the decimal interest on $181.25, for 2*y.* 4*mos.* 19*d.*, at 5 per cent.?

10. What is the decimal interest on $95. for 7*y.* 3*mos.* at 6 per cent.?

11. What is the amount of $5431, for 33 days, at 6 per cent., bank interest?

12. What is the bank interest of $47965, for 34 days, at 5 per cent.?

13. What is the exact amount of $10000, for 64 days, at 7 per cent.?

14. What is the decimal interest of $5387.75, from 10th mo. 3d, 1842, to 9th mo. 1st, 1844, at 4½ per cent.?

15. What is the amount of $99.50, from July 4th, 1839, to June 27th, 1843, at 6 per cent.?

16. What is the bank interest of $500.00, for 123 days, at 5 per cent.?

17. What is the bank interest of $2500, for 94 days, at 4 per cent.? What is the amount?

18. What is the amount of $730.00, from Dec. 1st, 1829, to Jan. 31st, 1835, at 7 per cent.?

19. What is the amount of $629.50, from 8th mo. 3d, 1840, to 7th mo. 2d, 1842?

20. Bought 75¾ pounds of tea, at 62½ cents per pound, for which I gave my note payable in 6 months, dated 6th mo. 5th, 1843. What was the amount of the note 3d mo. 27th, 1844? [Interest does not commence until the note becomes due.]

21. John Smith gave a note for $2000 to the Bank of North America, payable in 60 days. How much money did he receive, after deducting 63 days' interest?

22. How much can I receive on a note at 90 days, for $639.50, after deducting 93 days' interest?

23. What is the present worth of a 30 days' note for $1870, after deducting 33 days' interest?

24. What is the amount of a note for $2131.00, for 183 days, at 6 per cent., bank interest?

PRACTICE RULE.

Multiply 1 per cent. of the principal by $\frac{1}{2}$ the even number of months, and if there is an odd month, add 30 to the number of days. Divide the days by 6, and multiply $\frac{1}{10}$ of 1 per cent. by the quotient. If there are any remaining days,* take as many 60ths of 1 per cent. Add the numbers so obtained, and their sum will be the interest at 6 per cent. For any other rate, increase or diminish the result, as in the Bank Rule.

EXAMPLE FOR THE BOARD.

What is the interest of $9763.25 for 2*y*. 9*mo*. 8*d*.?

1 per cent. is	97.6325
$\frac{1}{2}$ of 2*y*. 8*mo*. is	16
	585.7950
	976.325
36*d*. is .6 of 2*mo*.	58.5795
2*d*. is $\frac{1}{30}$ of 2*mo*.	3.2544
	$1623.9539

1 per cent., or the interest for 2 months, is $97.6325. Multiplying by 16, we obtain the interest for 2*y*. 8*mo*. The 1*mo*. remaining, added to the 8 days, gives 38 days, in which 6 is contained 6 times, with 2 remainder. As the interest for 6 days is .1 of 1 per cent., or $9.763+, the interest for 36 days is 6 times as much. The 2 remaining days are $\frac{1}{30}$ of 60 days, we therefore add $\frac{1}{30}$ of $9.763+.

25. What is the interest of $281.50 for 3*y*. 7*mo*. 16*d*.? What is the amount?

26. What is the interest of $1175 for 1*y*. 11*mo*. 29*d*.? What is the amount?

27. What is the interest of $28.75 for 2*y*. 3*mo*. 4*d*., at 5 per cent.? What is the amount?

28. What is the interest of $3360 for 3*y*. 3*mo*. 3*d*., at 7 per cent? What is the amount?

29. What is the amount of $20000, from Sept. 30th, 1839, to March 27th, 1844, at 5 per cent.?

---

* When the remainder is 4, it is more convenient to diminish the quotient by 1, and call the remainder 10, taking $\frac{10}{60}$ or $\frac{1}{6}$ of 1 per cent. for the interest.

30. What is the amount of $531.50, from Nov. 3d, 1840, to Feb. 25th, 1842, at 7 per cent.?

31. What is the amount of $74.23, from June 29th, 1842, to Sept. 13th, 1844, at 4½ per cent.?

32. Sold Oct. 11th, 1830, 162*bu.* 3*pk.* of oats, at 37½ cents a bushel, and received in payment a note payable in 3 months. What was the amount of the note, May 5th, 1840?

PARTIAL PAYMENTS.

When partial payments have been made upon notes that are settled within a year from their date, it is customary to compute interest upon the whole note, and also on each payment, until the time of settlement, and deduct the amount of the payments from the amount of the principal. This is the *equitable* rule for all periods of time, but the following is the

LEGAL RULE.*

If any payment exceeds the interest due, deduct it from the amount, and compute interest on the balance. If the payment is less than the interest, add the excess to the succeeding interest, and continue the interest on the former principal.

EXAMPLE FOR THE BOARD.

$1000.00 Philadelphia, March 4th, 1841.

For value received, I promise to pay John Smith, or order, one thousand dollars on demand, without defalcation.

WILLIAM BROWN.

December 1st, 1841, received $75.00. July 17th, 1842, received $15.50. August 18th, 1843, received $30.50. December 11th, 1843, received $500.00. January 3d, 1844, received $150.00. What was due on the note, Aug. 18, 1844?

First principal, on interest from March 4, 1841.....	$1000.00
Interest to Dec. 1, 1841, (8*mo.* 27*d.*)..............	44.50
Amount,	$1044.50
First payment, exceeding the interest due, ...	75.00
Balance for a new principal	969.50

* See Johnson's Chancery Reports, Vol. I., p. 17.

New Principal		969.50
Interest from Dec. 1, 1841, to July 17, 1842, (7*mo.* 16*d.*)	36.52	
Second payment, less than interest due	15.50	
Excess of interest	21.02	
Interest from July 17, 1842, to Aug. 18, 1483, (13*mo.* 1*d.*)	63.18	
Interest due Aug. 18, 1843	84.20	
Third payment, less than interest due	30.50	
Excess of interest	53.70	
Interest from Aug. 18, 1843, to Dec. 11, 1843, (3*mo.* 23*d.*)	18.26	71.96
Amount due Dec. 11, 1843		1041.46
Fourth payment, exceeding the interest due		500.00
Balance for a new principal		541.46
Interest from Dec. 11, 1843, to Jan. 3, 1844, (23*d.*)		2.07
Amount due Jan. 3, 1844		543.53
Fifth payment, exceeding the interest due,		150.00
Balance for a new principal		393.53
Interest from Jan. 3, 1844, to Aug. 18, 1844 (7*mo.* 15*d.*)		14.76
Amount due Aug. 18, 1844		$408.29

1. Worcester, July 4th, 1840.

For value received, I promise to pay Andrew Jackson, or order, six hundred and thirty-nine dollars, on demand.

$639.00 JOHN WINTER.

Endorsements. Sept. 5, 1840, received $13.25. Jan. 1, 1841, received $1.50. March 17, 1841, received $72.00. Oct. 3, 1841, received $29.50. July 3, 1842, received $9.00. What was due Jan. 1, 1843?

2. Portland, May 13th, 1841.

For value received, I promise to pay George Appleton, or order, nine hundred dollars, on demand.

$900.00 WILLIAM MASON.

Endorsements. Aug. 28, 1843, received $175.00. Dec. 13, 1843, received $10.00. April 13, 1844, received $10.00. May 1, 1844, received $500.00. What was due Sept. 13, 1844?

3. New Orleans, April 3d, 1839.

For value received, I promise to pay Earle & Chase, or order, five hundred and twenty-five dollars and fifty cents, on demand.

$525.50 CHARLES CHAMPION.

Endorsements. April 13, 1840, received $29.75. Feb. 4, 1841, received 15.00. March 30, 1841, received $250.00. Jan. 1, 1842, received $175.75. What was due on the settlement of the note, April 3, 1844?

4. New York, Oct. 5th, 1823.

For value received, I promise to pay Brown & Oliver, or order, one thousand dollars on demand.

$1000.00 JAMES THOMAS.

Endorsements. March 4, 1824, received $100.00. July 27, 1825, received $50.00. Oct. 25, 1825, received $100.00. April 13, 1826, received $15.00. Nov. 13, 1826, received $10.00. Dec. 1, 1826, received $500.00. What was due on the note, May 16, 1830?

5. Charleston, Aug. 18th, 1840.

For value received, I promise to pay Nathan J. Wilson, or order, four hundred and thirty-one dollars [redacted], with interest [redacted].

$431.00 EDWARD ELLIS.

Endorsements. Feb. 18, 1841, received $31.00. Sept. 15, 1841, received $10.00. Nov. 11, 1841, received $5.00. March 29, 1842, received $100.00. May 13, 1843, received $200.00. Dec. 31, 1843, received $2.50. What was due June 16, 1844?

6. Cincinnati, Nov. 1st, 1841.

For value received, we promise to pay Samuel Jones, or order, seven hundred and seventy-five dollars and fifty cents, in two months from date.

$775.50 HENRY THOMPSON & Co.

Endorsements. Feb. 27, 1842, received $15.00. Aug. 20, 1842, received $15.00. Dec. 18, 1842, received $15.00. Jan. 1, 1843, received $15.00. April 1, 1843, received $200.00. Feb. 19, 1844, received $21.00. What was the balance due Oct. 1, 1844?

ACCOUNTS CURRENT.

In computing interest on accounts current, all charges are entered on the Dr. side, when they become due, and all receipts are entered on the Cr. side when they are paid. Such accounts are balanced annually, semi-annually, or quarterly. To facilitate the settlement, it is customary to place against each sum the number of days that will elapse till the time the books are balanced, and calculate the interest on each charge. Then *take the sum of the Dr. interests, and the sum of the Cr. interests; the difference of these two sums will be the balance of interest.*

EXAMPLE FOR THE BOARD.

Abbott & Clark, Philadelphia, in Account Current with Charles Goodhue & Co., Boston.

Dr.

1844		Dol. cts.	No. of days.	Interest.
June 1	To Balance due from former acct.,	153.50	92	2.354
" 13	To amount due on note for goods,	400.00	80	5.333
" 25	To merchandise,	275.00	68	3.116
July 19	To 90 bbls. flour, at $4.37½	393.75	44	2.887
Sept. 1	To balance of interest,	3.28		13.690
		$1225.53		10.412
				3.278

Cr.

1844		Dol. cts.	No. of days.	Interest.
June 16	By cash,	375.00	77	4.812
" 30	By bill of Arnold & Brown,	250.00	63	2.625
July 29	By cash,	525.00	34	2.975
Sept. 1	By balance to your debit on a new account,	75.53		10.412
		$1225.53		

$13.69—$10.41=$3.28, balance of interest due. The balance of the *account* is found by taking the difference between the Dr. and the Cr. side.

1. *Oliver Marriott, Savannah, in Account Current with Daniel Clark, Mobile.*

Dr.

1844		Dol. cts.	No. of days.	Interest.
Feb. 16	To balance due from old account,	91.75		
" 25	To merchandise,	163.50		
Apl. 13	To merchandise,	219.25		
May 16	To balance of interest,			

1844	*Cr.*	Dol. cts.	No. of days.	Interest.
Mar. 30	By cash,	400.00		
May 16	By balance to new acct.			

2. *Joseph Mason & Co., Saint Louis, in Account Current with Thompson & Brother, Lexington.*

Dr.

1844		Dol. cts.	No. of days.	Interest.
Jan. 13	To sheeting,	131.50		
Feb. 3	To duck,	87.75		
" 25	To cambric,	240.00		
Mar. 29	To sundries,	300.00		
Apl. 1	To balance of interest,			

1844	*Cr.*	Dol. cts.	No. of days.	Interest.
Jan. 29	By furs,	175.00		
Mar. 3	By bill,	200.00		
Mar. 18	By cash,	[illegible].75		
Apl. 1	By balance to new acct.			

3. *Henry Chatham, Nashville, in Account Current with George Hapgood & Sons, Baltimore.*

Dr.

1844		Dol. cts.	No. of days.	Interest.
Apl. 5	To amount due on note for goods,	500.00		
May 17	To ditto,	119.50		
" 28	To merchandise,	87.25		
June 16	To sundries,	63.00		
July 1	To balance to new acct.			

1844	*Cr.*	Dol. cts.	No. of days.	Interest.
June 9	By cash,	350.00		
" 11	By tobacco,	289.50		
" 23	By bill on Atkins & Jones,	200.00		
July 1	By balance of interest,			

COMPOUND INTEREST.

It is sometimes stipulated that the interest shall be paid quarterly, semi-annually or annually, and if not paid at the specified time, the interest is added to the principal, and the amount is regarded as a new principal, on which interest is computed to the time it again becomes due. Such interest is called COMPOUND INTEREST, but, though often charged, it is not strictly legal.

To find the amount of any sum at compound interest, *Increase the principal by the interest each time it becomes due, and consider the amount a new principal.*

EXAMPLE FOR THE BOARD.

What will be the amount of $280 for 1*y*. 10*mo*. 22*d*., interest payable semi-annually?

```
       280
       .03  rate for 6mo.
      8.40
      280
    288.40  amt. for 6mo.
       .03
    8.6520
    288.40
   297.052  amt. for 1y.
       .03
   8.91156
   297.052
 305.96356  amt. for 1y. 6mo.
     .0236  rate for 4mo. 22d.
7.220740016
305.96356
313.1843         amt. for 1y. 10mo. 22d.
```

The interest may be obtained by subtracting the principal from the amount. In the present example, the interest is $313.18—$280.00 =$33.18.

1. What is the amount of $100 for 1*y*. 2*mo*. 15*d*., at 5 per cent., interest payable quarterly?

2. What is the amount of $375 for 2*y*. 11*mo*. at 4½ per cent., interest payable semi-annually?

3. What is the compound interest of $180 for 4*y.* 5*mo.*?

4. What is the difference between the simple and compound interest of $1000, for 3*y.* 9*mo.* 12*d.*?

5. What is the compound interest of $50 for 1*y.*, interest payable monthly?

6. What is the amount of $225.50 for 1*y.* 11*mo.* 25*d.*, at 7 per cent., interest payable semi-annually?

7. A note for $631 was dated June 19, 1841. To what did it amount Aug. 25, 1844, the interest being payable semi-annually?

[For more examples in Compound Interest, see GEOMETRICAL PROGRESSION, and ANNUITIES.]

PROBLEMS IN INTEREST.

I. THE PRINCIPAL, INTEREST, AND TIME, BEING GIVEN, TO FIND THE RATE,—

Divide the given interest by the interest at 1 *per cent. for the given time.*

1. If I pay $68.705 for the use of $1963.00 six months, what is the rate per cent.?

2. At what rate per cent. will $421.50 amount to $674.40 in 6*y.* 8*mo.*? [The interest is found by subtracting the principal from the amount.]

3. At what rate per cent. will a merchant double his capital in six years?

4. If a capital of $913 amounts to $1250 in 15 months, what is the annual gain per cent.?

II. THE PRINCIPAL, INTEREST AND RATE, BEING GIVEN, TO FIND THE TIME,—

Divide the given interest by the interest for 1 *year, at the given rate.*

5. In what time will $1500, at 7 per cent., amount to $1920?

6. The interest on a note of $750 was $131.25, and the rate was .05. What was the time?

7. A note of $431.00 amounted, at its settlement, to $546.29¼. How long had it been on interest, the rate being 6 per cent.?

8. In what time will any principal be doubled, at 4 per cent.? At 5 per cent.? 6 per cent.? 7 per cent.?

III. THE TIME, RATE AND INTEREST, BEING GIVEN, TO FIND THE PRINCIPAL,—

Divide the given interest by the interest of \$1.00 *for the given time.*

9. What principal, at 6 per cent., will yield \$150 in 18 months?

10. A man's property yields 5½ per cent. interest. What is he worth, his annual income being \$1375?

11. The quarterly expenditure of a charity school is \$600. What sum of money, yielding 8 per cent., will support it?

12. What is the face of a note which, at 7 per cent., will yield \$111.65 interest in 3*y*. 8*mo.*?

IV. THE TIME, RATE AND AMOUNT, BEING GIVEN, TO FIND THE PRINCIPAL,—

Divide the given amount by the amount of \$1.00 *for the given time.*

13. What principal will amount to \$500.00 in 2*y*. 6*mo.*, —or what is the *present worth* of \$500.00, due in 2*y*. 6*mo.*?

14. What is the present worth of \$1844, due in 1*y*. 8*mo.*?

15. What is the present worth of \$725.50, due in 6*y*., interest at 5 per cent.?

16. What is the present worth of \$2000.00, due in 3*y*. 6*mo.*, interest at 7 per cent?

DISCOUNT.

DISCOUNT is an allowance for the payment of money before it is due. Equitable Discount is found by subtracting the present worth (which is found by Problem IV. of the last chapter) from the amount.

BANK, or BUSINESS DISCOUNT, is the same as Bank Interest. In computing discount, it is customary to add 3 days* to the time stated in the note. These 3 days are

* In Pennsylvania, 4 days are added.

called *days of grace*, and the note is not legally due until the last day of grace.

1. A note for $2000, payable in 30 days, was discounted at a bank. What was received for the note?

2. What is the bank discount on a note for $1750, payable in 60 days?

3. How much can I obtain by discounting each of the following notes, at 7 per cent.?

$375.50 Providence, Sept. 19, 1844.

Ninety days from date, I promise to pay Jacob Murray, or order, three hundred and seventy-five dollars and fifty cents, for value received. MORRIS JOHNSON.

$500.00 Hartford, Sept. 3, 1844.

Thirty days from date, I promise to pay Evans & Powell, or order, five hundred dollars, for value received.
JOHN MILTON.

$795.00 Nashville, Aug. 5, 1844.

Six months after date, we promise to pay Charles W. Taylor, or order, seven hundred and ninety-five dollars, for value received. EARLE & KING.

4. What are the present worth and the equitable discount of $1870, due in 3*y.* 4*mo.* 13*d.*?

5. Wilmington, 5th mo. 7th, 1844.

For value received, I promise to pay Jonathan Mason, or order, six hundred and seventy-three dollars and twenty-five cents, in six months.
MOSES A. WILSON.

$673.25

What ought Moses A. Wilson to pay, in order to cancel the above note, 8th mo. 3d, 1844?

6. The Farmers' and Mechanics' Bank offer to discount the following notes at 5 per cent. How much shall I receive on each?

$200.00 Philadelphia, 8th mo. 3d, 1844.

Sixty days from date, I promise to pay Pliny E. Chase, or order, two hundred dollars, for value received.

WILLIAM JACOBSON.

$144.00 Baltimore, Aug. 1, 1844.

Ninety days from date, I promise to pay Pliny E. Chase, or order, one hundred and forty-four dollars, for value received. GEORGE M. WATSON.

$75.75 New York, Sept. 5, 1844.

Six months from date, I promise to pay Pliny E. Chase, or order, seventy-five dollars and seventy-five cents, for value received. MARCUS GRAHAM.

7. What is the bank discount on a note of $1270 for 30 days? 60 days? 120 days?

8. What is the difference between the equitable and the bank discount on $2000 for 1 year?

9. How much do I gain by purchasing molasses for $1600 cash, and selling it immediately for $2000, payable in 9 months,—the People's Bank discounting the note at 7 per cent.?

10. Sold George Manning & Brothers, $6\frac{4}{5}$*cwt.* of sugar, at $5\frac{1}{2}$*cts.* a pound; 81 barrels of flour, at 4.62\frac{1}{2}$ per barrel; 3.679*cwt.* of coffee, at $8\frac{5}{8}$*cts.* a pound; 2*cwt.* 1*qr.* 17*lb.* of rice, at $3.50 per *cwt.*; 13 boxes sperm candles, each containing $32\frac{1}{2}$*lb.*, at $29\frac{3}{4}$*cts.* per *lb.*; and 19*lb.* 11*oz.* chocolate, at $18\frac{3}{4}$*cts.* a pound. How much was received on their note for the amount, at 90 days, discounted at the National Bank?

11. Sold Clark & Atkins, 3*cwt.* 1*qr.* 17*lb.* of sugar, at $5.00 per *cwt.*; $391\frac{3}{8}$*lb.* Young Hyson tea, at $87\frac{1}{2}$*cts.* a pound; $250\frac{1}{2}$*lb.* Imperial tea, at $63\frac{7}{8}$*cts.* a pound; 78*gal.* 3*qt.* of molasses, at $31\frac{1}{4}$*cts.* a gallon; and $127\frac{3}{8}$*lb.* cheese, at $8\frac{3}{4}$*cts.* a pound, receiving their note at 120 days for the amount. What is the value of the note, discounting at 7 per cent.?

12. What is the value of a note at 60 days, for the following amounts? $163\frac{5}{8}$*yd.* of silk, at $87\frac{1}{2}$*cts.* a yard,

$491\frac{1}{2}$*yd.* India muslins, at $62\frac{1}{2}$*cts.* a yard; $363\frac{3}{4}$*yd.* Alpacca, at $93\frac{3}{4}$*cts.* a yard; 224*yd.* merino, at 50*cts.* a yard; and $251\frac{3}{8}$*yd.* printed lawn, at $37\frac{1}{2}$*cts.* a yard.

13. What is the value of a note at 30 days, for $24491.75, discounting at $5\frac{1}{2}$ per cent.?

14. What is the value of a note at 60 days, for $1874.50, discounting at $4\frac{3}{4}$ per cent.?

15. What is the value of a note at 90 days, for $4959.375, discounting at $4\frac{7}{8}$ per cent.?

16. What is the value of $\frac{1}{7}$ of $\frac{3}{8}$ of a note at 120 days, for $7638, discounting at $5\frac{1}{4}$ per cent.?

17. What is the value of $\frac{1}{2}$ of $\frac{5}{9}$ of a note at 30 days, for $10000, discounting at $7\frac{1}{2}$ per cent.?

18. What is the value of $\frac{2}{3}$ of .97 of $\frac{8}{9}$ of a note at 4 months, for £2610 7*s.* 6*d.*, discounting at 5 per cent.?

19. A merchant has five notes at 60 days, of equal value, amounting in the whole to £$19763\frac{1}{3}$. What is the worth of .85 of one of them, in pounds, shillings and pence, discounting at $4\frac{1}{2}$ per cent.?

20. What is the value of a note at 6 months, for $1175.875, discounting at $8\frac{3}{4}$ per cent.?

CHAPTER X.

EQUATION OF PAYMENTS AND AVERAGE.

When several debts are due at different times, it is often desirable to find some time at which they may all be paid, without loss to either the debtor or the creditor. This time is called the AVERAGE, or EQUATED TIME, and the process by which it is found is the EQUATION OF PAYMENTS.

EXAMPLE FOR THE BOARD.

A merchant owes $4.00, payable in 3 months; $6.00 payable in 6 months, and $2.00 payable in a year. At what time may he pay the whole, so that neither party shall lose any interest?

The interest of \$4.00 for 3*mo.*=The int. of \$1.00 for 12*mo.*
The interest of \$6.00 for 6*mo.*=The int. of \$1.00 for 36*mo.*
The interest of \$2.00 for 12*mo.*=The int. of \$1.00 for 24*mo.*

The interest of \$12.00 for —*mo.*=The int. of \$1.00 for 72*mo.*

He may keep \$12.00, $\frac{1}{12}$ as long as \$1.00. Therefore, dividing 72*mo.* by 12, we obtain 6*mo.* for the equated time.

RULE.

Multiply each debt by the time in which it is payable, add all the products, and divide their sum by the sum of the debts.

This rule, which is universally adopted, is founded on the supposition that the interest of the money, which is not paid until after it is due, is equal to the discount of that which is paid before it is due. This is not strictly correct, but the difference is so trifling, that it does not materially affect the ordinary transactions of business.

1. I have three notes given by James Appleton; one of \$75, payable in 30 days, one of \$50, payable in 60 days, and one of \$500, payable in 90 days. What is the equated time for the payment of the whole?

2. A merchant sold goods, agreeing to receive $\frac{1}{2}$ of the amount in 3 months, $\frac{1}{3}$ in 6 months, and the rest in 9 months. At what time might the bill be settled by a single payment?

3. What is the equated time for the payment of \$1000, $\frac{1}{4}$ of the sum being now due, $\frac{1}{4}$ in 3 months, and $\frac{1}{2}$ in 9 months? [The \$250 now due has no time to run, and the product $250\times0=0$.]

4. What is the equated time for the payment of a debt, $\frac{1}{4}$ of which is now due, $\frac{1}{3}$ in 3 months, $\frac{1}{6}$ in 9 months, and the remainder in 15 months?

5. What is the equated time for the payment of three debts, one of \$100, due in 8*mo.*, one of \$25, due in $1\frac{1}{4}$*y.*, and one of \$90, due in 2*y.*?

6. On a debt of \$400, due in 9*mo.*, a payment of \$100 is made in 3*mo.* In what time should the remainder be paid?

7. I owe \$40, payable now, \$40 in 30 days, \$20 in 60

days, and $100 in 120 days. What is the equated time for the payment of the whole?

8. Manson & Hill, of Liverpool, have given to Thomas Morton & Co. of New York, a note of £225 10*s.*, payable in 60 days; one of £196, payable in 60 days; one of £218 7*s.* 6*d.*, payable in 90 days; and one of £300, payable in 120 days. At what time may the notes all be cancelled by a single payment?

9. On a debt of £1100, due in 9*mo.*, a payment of £159 4*s.* is made in 3*mo.*, and another of £300 in 4½*mo.* What is the equated time for the payment of the balance?

By this rule we may obtain the average price of a number of ingredients of different value. The quantity of the several ingredients is substituted for the debts, and the price for the times.

10. A grocer mixes 10 pounds of tea, at 50 cents a pound, 12 pounds at 75 cents, and 16 pounds at 62½ cents. What is the value of a pound of the mixture?

11. If 12 gallons of water are mixed with 9 gallons of wine, at $1.00 per gallon, and 12 gallons at $1.50, what is the mixture worth per gallon?

12. Required the average length of 5 pieces of sheeting, measuring as follows: No. 1, 32⅛*yd.*; No. 2, 31½*yd.*; No. 3, 33½*yd.*; No. 4, 30¾*yd.*; No. 5, 34*yd.*

13. If I mix 3*lb.* of gold 22 carats fine, 5*lb.* 20 carats fine, 8*lb.* 24 carats fine, 10*lb.* 23 carats fine, and 2*lb.* of alloy, what will be the fineness of the mixture? [A carat is $\frac{1}{24}$. Pure gold is 24 carats fine. Alloy is counted of no value.]

14. A grocer mixed 18 gallons of molasses, at 25 cents a gallon, with 20 gallons at 30 cents, 24 gallons at 40 cents, and 5 gallons of water. At what price per gallon must he sell the mixture to gain 25 per cent.?

15. If the thermometer stands from 6 to 9 at 68°, from 9 to 11 at 70,° from 11 to 1 at 73°, from 1 to 4 at 77°, and from 4 to 6 at 76°, what is the average temperature of the 12 hours?

16. The following charges are made against Walton & Morris, each payable in 8 months, viz: Jan. 25, $1643.50; Feb. 1, $900; Feb. 16, $1131.25; Feb. 28, $241.

75; March 13, \$971; and April 3, \$763.37. What should be the date of a single note at 8 months, for the whole amount?

17. What must be the date of a note at 6 months, for the amount of the following charges, each payable in 8 months? March 19, \$59.63; April 3, \$713.20; April 15, \$2350; April 21, \$1975.37; July 5, \$163.25; and Sept. 19, \$251.17.

18. What should be the date of a note at 9 months, for the amount of the following charges? May 23, \$840 at 6*mo.*; May 30, \$900 at 8*mo.*; June 15, \$325 at 4*mo.*; June 27, \$563.50 at 3*mo.*; July 13, \$729 at 6*mo.*; and July 16, \$421.75 at 9*mo.*

19. A jeweller mixed 3*lb.* 6*oz.* silver, 9*oz.* fine, 4*lb.* of 11*oz.* fine, 2*lb.* 3*oz.* of 10½*oz.* fine, and 2*lb.* of 10*oz.* fine. What was the fineness of the mixture? [11*oz.* fine, denotes that 11*oz.* in the *lb.* are silver, and the rest alloy. Pure silver is 12*oz.* fine.]

20. If 1*lb.* silver, 8*oz.* fine, 2*lb.* of 9*oz.* fine, 3*lb.* of 10*oz.* fine, 4*lb.* of 11*oz.* fine, and 5*lb.* alloy, be melted together, what will be the fineness of the mixture?

21. Marshall & Wood, to George W. Hall, Dr.

1844		
4 mo. 16,	To 143½*yds.* sheeting, at 12¼*cts.*	8*mo.*
" 23,	To 259¾*yds.* " at 11¾*cts.*	8*mo.*
5 mo. 2,	To 169*yds.* cambric, at 13*cts.*	6*mo.*
"	231¼*yds.* bleached muslin, at 13⅝*cts.*	4*mo.*
5 mo. 9,	To 258⅝*yds.* duck, at 11½*cts.*	6*mo.*

What should be the date of a note at 6*mo.* for the whole amount?

22. A farmer mixed 18½ bushels of wheat, at \$1.00 per bushel; 16¾*bu.* at \$1.12½ per bushel; 13⅜*bu.* of barley, at 62½*cts.* per bushel, and 10*bu.* of oats, at 37½*cts.* per bushel. What was the mixture worth per peck?

23. What should be the date of a note at 60 days (or 63 days, including the days of grace) for \$150, due April 7,—\$200, due April 13,—\$75, due April 17,—\$325, due April 29,—\$180, due April 30,—\$400, due May 4,—and \$439, due May 11?

CHAPTER XI.

PROPORTION, OR THE RULE OF THREE.

The Ratio of two numbers is the quotient resulting from the division of the first by the second. Thus, the ratio of 1 to 2, is $\frac{1}{2}$; of 13 to 6, $\frac{13}{6}$.

A ratio is usually expressed by two points written between the numbers, as 1 : 2; 13 : 6; which are read 1 *is to* 2; 13 *is to* 6.

A Proportion is the union of two equal ratios, by writing four points, or the sign of equality, between them. Thus, 2 : 3 :: 4 : 6, or 2 : 3=4 : 6, is a Proportion, and may be read, 2 *is to* 3 *as* 4 *is to* 6. The numbers themselves are called Proportionals.

The first term of every ratio is called the *antecedent*, and the second the *consequent*. The first and fourth terms are called the *extremes*, the second and third terms the *means*, and in every proportion *the product of the extremes is equal to the product of the means*. Take for example the proportion 3 : 9 :: 5 : 15. This may also be written $\frac{3}{9}=\frac{5}{15}$. Reducing these fractions to a common denominator, we have $\frac{45}{135}=\frac{45}{135}$. The numerators are now the same, but one is the product of the extremes, and the other the product of the means.

The antecedents and consequents may, therefore, change places in a variety of ways, the proportion always continuing so long as the product of the means is equal to the product of the extremes.

Then, whenever one of the extremes and the two means are given, to find the other extreme, *Divide the product of the means by the given extreme.*

If any number will divide either antecedent and its consequent, the division may be performed without destroying the proportion. Thus, the proportion 9 : 12 :: 15 : 20 is the same as $\frac{9}{12}=\frac{15}{20}$. Dividing 9 and 12 by 3, or 15 and 20 by 5, the corresponding fraction will be reduced to its lowest terms, and the result, 3 : 4 :: 15 : 20, or 9 : 12 :: 3 : 4, is a proportion.

There are many other interesting properties of proportions, which belong more properly to the province of

Geometry. What we have already learned, is sufficient for all Arithmetical purposes.

EXAMPLES FOR THE BOARD.

If 8 men build 40 rods of wall in 3 days, how many men will build 100 rods in 12 days?

To build 100 rods it will evidently take $\frac{100}{40}$ as many men as to build 40 rods, or $\frac{100}{40}$ of 8, which may be expressed by the proportion

rods.		rods.		men.		men.
40	:	100	::	8	:	—

To build the wall in 12 days, it will require but $\frac{3}{12}$ as many men as to build it in 3 days, which may be expressed by the proportion

days.		days.		men.		men.
12	:	3	::	$\frac{100}{40}$ of 8	:	—

These two proportions may be written together, thus,

$\left.\begin{array}{r} 40 : 100 \\ 12 : 3 \end{array}\right\} :: \overset{\text{men.}}{8} : \overset{\text{men.}}{—}$	Or by cancelling the like factors in the antecedents and consequents,	$\left.\begin{array}{r} 2 : 5 \\ 4 : 1 \end{array}\right\} :: 8 : —$
$480 : 300 \quad :: 8 : —$		$8 : 5 \quad :: 8 : —$

Multiplying together the two antecedents and the two consequents of the first ratio, we obtain a single proportion, in which the two means and one extreme are given. Dividing the product of the means by the given extreme, we obtain 5 men for the answer, and our completed proportion is,

				men.		men.	
8	:	5	::	8	:	5	*Ans.*

Hence we derive the RULE OF THREE.

Write that number which is of the same kind as the answer, for the third term of a proportion. Of the remaining quantities, compare any two of a kind, and consider whether the answer will be greater or less than the third term. If greater, write the greater number for the second term, and the less for the first. If less, write the less number for the second term, and the greater for the first. In the same way compare any other two of a kind, and so proceed until all the numbers are employed. Cancel the like factors in the antecedents and consequents, and for the fourth term, or answer, multiply the third term by the product of the second terms, and divide by the product of the first terms.

N.B. Before cancelling, all compound numbers must be reduced to the same denomination.

The pupil should be required, after performing the ex-

amples in this chapter according to the rule, to give an analytical solution of each question.

1. State 274×16 in the form of a proportion, by making 1 the first term.

2. State $6517 \div 19$ in the form of a proportion, by making 1 either the second or third term.

3. What is the value in dollars and cents of the following Bill of Exchange, — exchange at 5*fr.* 32*c.* per dollar?

Exchange 10000*fr.* **Second.** Phil. 13th July, 1844.

Sixty days after sight of this, our second of Exchange (first and third of same tenor and date unpaid), pay to the order of J. Villiers & Cie., ten thousand francs, for value received, as per advice, for the account of

To Messrs. Noros & Chopin, Paris. THOMPSON & CLARK.

4. In 9450 milrees, 320 rees, how many dollars and cents?

5. Reduce 487 roubles, 63 copecks, to Federal money; exchange at 75 cents per rouble.

6. If 29 men can do a piece of work in 11 days, by working 10 hours a day, how long will it take 18 men, working 8 hours a day, to do the same work?

7. If 93 barrels of flour cost \$418.50, what will 137 barrels cost?

8. When exchange on London is at par, and £9=\$40, what is the value of £27 13*s.* 11*d.*?

9. How many men will reap 96 acres in 12 days, if 13 men reap 78 acres in 6 days?

10. If I pay \$14 for the freight of 3 tons, 45 miles, what must I pay for the freight of 11 tons, 17 miles?

11. What is the value of 19 marcs 12 schillings Hamburg,—exchange at 35 cents per marc?

It will readily be seen that all the terms of a proportion may be distinguished into multipliers and divisors. If, then, we write all the multipliers in one column, and all the divisors in another, and cancel the factors common to both columns, the answer may be obtained by dividing the product of the multipliers by the product of the divisors.

The terms of every proportion may also be distinguished into causes and effects, and the two products of each cause by its opposite effect are equal. For example, if 8 men build 4 rods of wall in a day, 4 men will build 2 rods in the same time. Stating the proportion, we have

$$\underset{\text{cause.}}{\overset{\text{men.}}{8}} : \underset{\text{cause.}}{\overset{\text{men.}}{4}} :: \underset{\text{effect.}}{\overset{\text{rod.}}{4}} : \underset{\text{effect.}}{\overset{\text{rod.}}{2}}$$

The product of the extremes, and the product of the means, give us the product of each cause by its opposite effect.

Times are causes, for 2 days will produce twice as much as one day.

In questions of freight, we may regard distances and bulk as causes, producing money for their effect.

A little practice will give great facility in making this distinction in all cases. If, then, we write each effect opposite to its cause, our multipliers and divisors will be obtained without difficulty.

EXAMPLE FOR THE BOARD.

If 18 men, in 6 days of 8 hours, build a wall 150 feet long, 2 feet wide and 4 feet high, in how many days of 12 hours will 24 men build a wall 200 feet long, 3 feet wide and 6 feet high?

men $\cancel{18}$	$\cancel{24}$ men.	causes.
days $\cancel{6}$	— days.	
hours $\cancel{8}$.	$\cancel{12}$ hours.	
long $\cancel{200}$	$\cancel{150}$ long.	effects.
wide 3	$\cancel{2}$ wide.	
high 3 $\cancel{6}$	$\cancel{4}$ high.	

9 days Ans.

Commencing our statements, we write 18 men, 6 days of 8 hours, as cause, and the effect, which is a wall 150*ft.* long, 2*ft.* wide, and 4*ft.* high, on the opposite side: opposite to each of these terms, we write — days, 12 hours, 24 men, as cause,— and 200 long, 3 wide, 6 high, as effect. Cancelling the like factors, we have but 3×3 on the side of the multipliers, and 1 on that of the divisors. $3\times3\div1$ is therefore the missing term, or number of days required. If either term were fractional, the denominator representing a divisor, should be transposed to the opposite side. By proceeding in this manner, a statement may be made as soon as the question can be proposed.

RULE.

Place the causes of the first set of quantities on one side of a vertical line, and the effects on the opposite side, and the causes and effects of the second set opposite to the corresponding quantities of the first, supplying the deficient term by a dash. Cancel the like factors on the opposite sides of the lines, and divide the product of the numbers opposite to the dash, by the product in which the dash is included. The quotient will be the term required.

12. How many men, in 6 months, will build a wall that 36 men will build in 8 months?

13. How many bushels of meal will serve 36 persons 12 months, if 10 persons consume 8 bushels in 2 months?

14. If 18 men build a cistern 20 feet long, 12 feet wide, and 10 feet high, in 3 weeks, by working 5 days in a week, and 9 hours a day, how many men, working 6 days in a week, and 12 hours a day, will build a cistern 32 feet long, 16 feet wide, and 15 feet high, in 12 weeks?

15. If \$12.00 gain \$3.00 in 5 months, how much ought \$25.00 to gain in 10 months?

16. If the freight of 900*lb.* for 56 miles, is \$2.50, how far may 2 tons be carried for \$40.00?

17. If \$1200 will support 24 persons 8 months, how long will \$900 support 16 persons?

EXAMPLE FOR THE BOARD.

If \$$27\frac{1}{2}$ buy $4\frac{3}{4}$ yards of cloth that is $\frac{5}{8}$*yd.* wide, how many yards of like quality, that is $\frac{5}{4}$*yd.* wide, may be bought for \$$13\frac{3}{4}$?

STATEMENT.

$27\frac{1}{2}$	$13\frac{3}{4}$
—	$4\frac{3}{4}$
$\frac{5}{4}$	$\frac{5}{8}$

Reducing the mixed numbers to improper fractions, we transpose the denominators, writing them above the causes,—then cancel and divide as before.

8		denom's.
$\cancel{4}$	$\cancel{4}$	
$2\cancel{4}$	$\cancel{2}$	
$\cancel{5}\cancel{5}$	$\cancel{5}\cancel{5}$	
—	19	
$\cancel{5}$	$\cancel{5}$	

Ans. $\frac{19}{16}=1\frac{3}{16}$*yd.*

18. If $57\frac{1}{5}$*cwt.* of sugar cost \$375, what will $18\frac{1}{2}$*cwt.* cost?

19. If the rent of 19*A*. 3*R*. of land is £4 10*s*., what will be the rent of $24\frac{1}{2}A$.?

20. The shadow of a stick that is 5 feet 6 inches high, measures 3*ft*. 4*in*. What is the height of a tree whose shadow measures 75 feet at the same time?

21. If the expenses of a family of 8 persons are $40 in 10 weeks, how many persons can be supported $12\frac{1}{2}$ weeks for $100?

22. How much wheat, at $1.20 a bushel, must be given in exchange for 90 barrels of flour, at $4.75 per barrel?

23. If 10 compositors, in 4 days of 10 hours, set $66\frac{2}{3}$ pages of types, each page containing 45 lines of 50 letters, how many compositors will set $94\frac{2}{7}$ pages, each page containing 35 lines of 40 letters, in $5\frac{1}{2}$ days of 8 hours?

24. If $29\frac{1}{3}$ bushels of wheat yield 1760 bushels in 5 years, how much will $15\frac{1}{2}$ bushels yield in 6 years?

25. A crew of 150 men were supplied with provisions for 9 months, allowing each man 2 pounds per day. When they have been out half the time, they find it will require 6 months to finish their voyage. How much may be allowed to each man, 25 of the crew having been lost?

EXAMPLE FOR THE BOARD.

A French merchant wishes to pay in London a bill of £1500. How many francs must he pay to procure remittances through Russia, Hamburg and Spain, allowing £13=75 roubles, 5 roubles=9 marcs of Hamburg; 3 marcs=1 Spanish dollar; and 9 dollars=50 francs?

Merchants often find an advantage in remitting bills circuitously, rather than directly to the place where they are due. The solution of such questions as the above, is called ARBITRATION OF EXCHANGE, and is best determined by THE CHAIN RULE, which is essentially the same as the Rule of Proportion.

£	$13=\cancel{75}\,5$	roubles.
rou.	$\cancel{5}=\cancel{9}$	marcs.
marcs	$\cancel{3}=1$	dollar.
dol.	$\cancel{9}=50$	francs.
francs	$—=1500$	£
	13) 375000	
	$28846fr.\ 15\frac{5}{13}c.$	

We write the quantities which are equivalent to each other, as antecedent and consequent, or as cause and effect, *making each effect of the same denomination with the next cause.* The like factors on opposite sides are cancelled, and the products divided as in proportion, to obtain the answer.

	£13	1500£
rou.	5	
mar.	3	
$	9	
		75 rou.
		9 mar.
		1 $
fr.	—	50 fr.

The question may be otherwise stated in the following manner: If £13 produce 75 roubles, 5 roubles produce 9 marcs, 3 marcs produce $1.00, and $9.00 produce 50 francs, how many francs will £1500 produce? The second set, or set of demand, contains but a single cause and effect. The first, or given set, contains a number of causes and effects, but they are so connected, that all the terms may be multiplied together, as a single compound term. Thus, if £13 produce 75 roubles, and 5 roubles produce 9 marcs, £13 will produce $\frac{75}{5}$ of 9 marcs, and $£13\times5$ will produce 75×9 marcs. In the same way it may be shown that $£13\times5\times3=\$75\times9\times1$, and $£13\times5\times3\times9=75\times9\times1\times50$ francs. Then, how many francs will £1500 produce?

26. A London merchant wishing to pay 1000 milrees in Lisbon, remits as follows: to Amsterdam at 36 schillings 7 groats per £; thence to Cadiz, at 17 groats for 2 rials of plate; thence to Leghorn, at 17 pezze for 100 rials; thence to Lisbon, at 1497 rees for 2 pezze. How many pounds did he remit?

27. If a merchant of New York remits $5000 to Havre, at 5*fr.* 35*c.* for $1.00; thence to London, at 49*fr.* for £2; thence to Hamburg, at 1 marc for 1*s.* 6*d.*; and thence to St. Petersburg, at 8 roubles for 17 marcs, how many roubles can he pay with his remittance?

28. If 33 copecks are equal to 5 English pence, 11 English pence are equal to 3 piasters, 13 piasters are equal to 1 florin, and 5 florins are equal to 29 francs, how many francs are equal to 9000 copecks? ×

29. If a man receives $30 for building 8 rods of wall, and he can purchase 3 barrels of flour for $14, and 3*cwt.* of sugar for 4 barrels of flour, and 21*lb.* of tea for 2*cwt.* of sugar, how many pounds of tea could he purchase by building 17 rods of wall?

30. If $1000 gain $\$11\frac{1}{9}$ in 80 days, how much will $2500 gain in 120 days, at the same rate?

31. If 13 days' work will purchase 1 hogshead of molasses, and 2 hogsheads of molasses are worth 5 tons of hay, and 3 tons of hay are worth 4 bags of coffee, how many bags of coffee can be bought with 39 days' labour?

32. If a man, by walking 3 miles an hour, for 6 hours a day, can accomplish a journey in 12 days, in how many days would a man walk the same distance, at the rate of $2\frac{1}{2}$ miles an hour, for 9 hours a day?

33. If $42\frac{1}{2}$ bushels of corn, that weighs $51\frac{1}{4}$ pounds a bushel, can be bought with 23 bushels of wheat, that weighs $56\frac{3}{4}$ pounds a bushel, how much corn, weighing 60 pounds a bushel, would be equivalent to 100 bushels of wheat that weighs 54 pounds a bushel?

34. If a man travels 240 miles in 8 days, when the days are 12 hours long, how many miles will he travel in 24 days, when the days are 16 hours long?

35. If the freight of 2*T.* 6*cwt.* for 28 miles, is \$14.50, what will be the freight of 9*T.* 4*cwt.* for 96 miles?

36. If 4 men in 3 days of 8 hours, build 40 rods of wall, how many rods will 18 men build in 5 days of 9 hours?

37. If I pay \$9.75 for carrying $2\frac{3}{5}$ tons 37 miles, how much ought I to pay for carrying 7*T.* 3*cwt.* 2*qr.*, 45 miles.

38. How many men, in 24 days of 16 hours, will do three times as much work as 18 men can perform in 36 days of 12 hours?

39. If \$495 will support a family of 7 persons, 11 months and 6 days, how much will maintain a family of 9 persons, for 13 months and 10 days?

40. How many persons may be supported a year, with \$1014, if \$140.25 will support 11 persons $8\frac{1}{2}$ weeks?

41. If \$973.16 yields an interest of \$91.25 in $13\frac{1}{2}$ months, what will be the interest of \$2801 for $17\frac{2}{3}$ months, at the same rate?

42. If $3\frac{1}{2}$ tons are carried $87\frac{1}{2}$ miles for $\$15\frac{1}{3}$, how far may $2\frac{7}{8}$ tons be carried for \$27.75?

43. If 11 men reap 16 acres of grain in $3\frac{1}{2}$ days, how many acres can 7 men reap in 15 days?

44. If 14 men, in $5\frac{1}{2}$ days, by working 8 hours a day, reap $38\frac{1}{2}$ acres of grain, how many men will reap $37\frac{1}{2}$ acres in $6\frac{3}{4}$ days, by working 9 hours a day?

45. If 12 men dig a trench 124*yd.* long, 4*ft.* wide, and

2*ft.* deep, in 6 days, by working 10 hours a day, how long a trench that is 7*ft.* wide, and 4*ft.* deep, will 45 men dig in 21 days, working $7\frac{1}{2}$ hours a day?

46. If $14\frac{5}{8}$ yards of cloth, that is $\frac{7}{8}$*yd.* wide, cost $29.50, what should be the price of $21\frac{1}{3}$*yds.* of similar quality, that is $1\frac{1}{2}$*yd.* wide?

47. If 300 tiles that are 9*in.* long and 6*in.* wide, will pave a court-yard, how many tiles would be required that are 6*in.* long and 4*in.* wide?

48. What is the weight of 16 iron bars, each 7*ft.* long, 6*in.* wide, and $3\frac{1}{2}$*in.* thick, if a bar 2*ft.* long, 2*in.* wide, and 1*in.* thick, weighs 18 pounds?

49. How many men will build a wall 240*yd.* long, 6*ft.* high, and 3*ft.* thick, in 8 days of 9 hours, if 7 men can build a wall 40*yd.* long, 4*ft.* high, and 2*ft.* thick, in 32 days of 7 hours?

50. If 70 braces of Venice are equal to 75 braces of Leghorn, and 7 braces of Leghorn are equal to 4 yards, how many yards are there in 79.375 braces of Venice?

51. A merchant in New York orders £500 sterling, due him in London, to be sent by the following circuit: to Hamburg, at 15 marcs banco per £; thence to Copenhagen, at 100 marcs banco for 33 rix-dollars; thence to Bourdeaux, at 3 rix-dollars for 18 francs; thence to Lisbon, at 125 francs for 18 milrees; and thence to New York, at $1.25 per milree. What was the arbitrated value of a dollar by this remittance?

52. If the freight of 11 boxes of sugar, each weighing 7*cwt.* 3*qr.* 11*lb.*, is $37.50 for 90 miles, what must be paid for the freight of 34 boxes, each weighing $8\frac{1}{18}$*cwt.*, for 75 miles?

53. How much wheat, that weighs 60*lb.* per bushel, would be required to supply a garrison of 1400 men 9 months, if 2800 bushels, weighing 58*lb.* per bushel, supply 800 men $3\frac{1}{2}$ months?

54. How many hours a day must 15 men work, to dig a trench 400*ft.* long, 6*ft.* wide, and 3*ft.* deep, in $187\frac{1}{2}$ days, if 72 men can dig a trench 250*ft.* long, 8*ft.* wide, and 4*ft.* deep, in $31\frac{1}{4}$ days, by working 7 hours a day?

55. If 16 men build a wall 30*ft.* long, 8*ft.* high and 3*ft.* thick, in 15 days, by working 9 hours a day, in how many days will 28 men, working 12 hours a day, build a wall 45*ft.* long, 6*ft.* high, and 2*ft.* thick?

56. If the wages of 9 men who work 7 hours a day, are $157.50 for 20 days, how many men will work 8 days, and 10 hours a day, for $130?

57. How many men can be furnished with 4 suits each, by 1140 yards of cloth that is $1\frac{1}{2}$*yd.* wide, if 2016 yards, $\frac{3}{4}$*yd.* wide, furnish 112 men 3 suits apiece?

58. How many days, travelling 6 hours a day, will be required to perform a journey of 1250 miles, if a journey of 900 miles can be accomplished in 12 days, by travelling 9 hours a day?

59. The provisions in a garrison are sufficient to support 800 men $3\frac{3}{4}$ months. How long would they supply 1500 men with but $\frac{2}{3}$ of the usual daily allowance?

60. If it costs $95 to furnish 78 men each with $\frac{3}{4}$*lb.* of bread daily, for 18 days, what will it cost to furnish to each of 93 men $1\frac{1}{2}$*lb.* daily, for 31 days?

61. If 18 men eat $2.50 worth of bread in 6 days, when wheat is $1.00 a bushel, in what time will 30 men eat $3.75 worth, when wheat is $1.10 a bushel?

62. If 16 compositors set 150 pages of types, each page containing 48 lines, and each line 50 letters, in 3 days of 10 hours, how many compositors will be required to set 500 pages of 72 lines each, and 45 letters in a line, in 6 days of 8 hours?

63. If 29 compositors set 300 pages of 40 lines, with 35 letters in a line, in $3\frac{1}{2}$ days of 8 hours, how many pages of 60 lines, and 50 letters in a line, will 33 compositors set in 11 days of 9 hours?

64. A wall which is to be built to the height of 27 feet, has been raised 9 feet in 6 days, by 12 men working 13 hours a day. How many men must be employed to finish it in 2 days, working only 12 hours a day?

65. If 4 men can saw 15 cords of oak in the same time that 5 men saw 14 cords of hickory, and if 3 men saw 18 cords of hickory in 3 days, by working 9 hours a day, how

many hours a day must 7 men work, to saw 84 cords of oak in 6 days?

66. If \$1700, at 6 per cent., yield an interest of \$350 in a given time, what will be the interest of \$3900 at 7 per cent. for one-half the time?

67. If 36 boxes, each containing 500 dates, support 27 men 24 weeks, how many boxes, each containing 350 dates, will support 72 men 48 weeks?

68. If 30 reams will print 1500 pamphlets of 10 sheets each, how many reams will print 740 pamphlets of $12\frac{1}{2}$ sheets each?

69. If 939 men drink 3510 quarts of water in 16.8 days, how many men will drink 1755 gallons in 3.36 days?

70. If $10\frac{3}{5}$ bushels of grain supply 11 horses 5.5 days, how long will 16 bushels supply 9 horses?

71. If \$25.75 will buy $20\frac{3}{5}$ bushels of wheat, how many bushels can be bought for \$$17\frac{5}{8}$?

72. When a post $11\frac{1}{4}$ feet high casts a shadow of $7\frac{3}{4}$ feet, what must be the height of a post that casts a shadow of 16 feet?

73. If 11*cwt.* 3*qr.* 17*lb.* of sugar cost £14 11*s.* 11*d.*, what will be the price of $7\frac{17}{30}$*cwt.*?

74. If $29\frac{1}{8}$*cwt.* of beef, cost \$87.50, how many pounds may be bought for \$31.75?

75. If 17 yards of serge, that is $\frac{3}{4}$ wide, are required to line a cloak containing 9.75 yards of cloth that is $5\frac{1}{3}$ quarters wide, how many yards that is a yard wide, would line a cloak containing 10.5 yards of cloth $6\frac{1}{8}$ quarters wide?

76. If it takes 7 men $9\frac{1}{2}$ days to perform a piece of work, working $7\frac{2}{3}$ hours a day, how long will it take 11 men to accomplish twice as much, working $8\frac{1}{2}$ hours a day?

77. If 16 men lay $35\frac{1}{2}$ rods of wall in 8 days, how many men will lay $53\frac{1}{4}$ rods in 6 days?

78. If a cistern discharge $83\frac{1}{3}$ gallons of water in 1.3 hours, how much will it discharge in $6\frac{4}{7}$ hours?

79. How far would a railway train move in 4.63 hours, at the rate of $47\frac{5}{6}$ miles in 3.15 hours?

80. What will be the price of $37\frac{7}{8}$ bushels of salt, if $13\frac{2}{3}$ bushels cost $11.75?

81. Amsterdam exchanges with London, at 34 schillings 4 pfennings per £, and with Lisbon at 52 pfennings for 400 reas. What is the arbitrated exchange between London and Lisbon, by way of Amsterdam?

82. When exchange on London is at a premium of $9\frac{1}{2}$ per cent., what is the value of $1863.50, in English money, the par of exchange being £1=4.44\frac{4}{9}$?

83. When exchange on London is at a premium of $8\frac{3}{4}$ per cent., what is the value in Federal money of £7963 8*s.* 3*d.*?

84. The exchange between New York and London is $4.84 per pound; between London and Amsterdam, 35 schillings per pound; between Amsterdam and Paris, 58 groats for 6 francs; between Paris and Venice, 10 francs per ducat; and between Venice and Cadiz at 360 maravedis per ducat. How many maravedis will be equivalent to $4500, by this circuitous remittance?

85. If 100*lb.* of Amsterdam are equal to 105*lb.* of Antwerp, 100*lb.* of Antwerp to 142*lb.* of Genoa, 100*lb.* of Genoa to 70*lb.* of Leipsic, and 100*lb.* of Leipsic to 104*lb.* of America, how many *lb.* of Amsterdam are equal to 759*lb.* of America?

86. Four men, working 13 hours a day, can do $\frac{5}{7}$ of a piece of work in 8 days: how many hours a day must 3 men work, to do $\frac{1}{2}$ the work in 11 days?

87. If 16 apples be worth 18 pears, and 4 pears can be bought for a cent, what will 180 apples cost?

88. If 9 men, in 21 days of 8 hours, mow 150 acres of meadow, in how many days of 9 hours can 11 men mow 36 acres?

89. If a garrison of 300 men eat 500 bushels of wheat in 2 months, how many bushels will 750 men eat in $1\frac{1}{2}$ months?

90. If I pay $40 for the carriage of $5\frac{3}{4}$*cwt.* 150 miles, what must I pay for carrying $7\frac{5}{7}$*cwt.* 64 miles?

91. If 9 suits of clothes can be made with $34\frac{3}{4}$ yards of cloth that is $1\frac{1}{2}$*yd.* wide, how many yards that is $\frac{7}{8}$ wide will make 17 suits of the same size?

92. If $2\frac{1}{2}$ yards of cloth $1\frac{3}{4}$*yd.* wide, will make 1 suit of clothes, how many yards $1\frac{1}{2}$*yd.* wide, will be required to clothe a garrison of 750 men?

93. If a 4 cent loaf weigh 8*oz.* when wheat is $1.25 per bushel, what should be the weight of a 6 cent loaf, when wheat is $1.00 per bushel?

94. The first term of a proportion is 7, the third term 12, and the product of the mean terms is $49\frac{1}{2}$. What is the difference between the third and fourth terms?

95. A may-pole 30 feet long, casts a shadow of 98 feet at a certain time. What is the width of a river, running at the foot of a tower 360 feet high,—the shadow of the tower at the same time reaching the opposite bank?

96. If the wages of 31 men for $4\frac{1}{4}$ days are $127.50, what will be the wages of 11 men for 8.5 days?

97. If 29 horses eat $153\frac{3}{4}$ bushels of oats in $1\frac{1}{2}$ months, how many bushels will 18 horses eat in 7 months?

98. If $72\frac{1}{3}$ acres will pasture 35 oxen 14.2 weeks, how long will $18\frac{1}{2}$ acres pasture 17 oxen?

99. In how many hours will a horse travel 84 miles, if by travelling $8\frac{1}{2}$ hours a day for $4\frac{1}{2}$ days, he goes 180 miles?

100. A vessel has provisions for 50 men $4\frac{1}{2}$ months, allowing each man $1\frac{1}{2}$*lb.* per day. How long would the same provision furnish 75 men, allowing 14*oz.* per day to each?

101. How much wall that is $2\frac{1}{2}$ feet thick, and 8 feet high, can 150 men build in 13 days of 9 hours, if 90 men, in 10 days of 10 hours, build 11 rods that is 2 feet thick and 6 feet high?

102. If 13 barrels of flour, at 3 cents a pound, cost $96.44, what will 27 barrels cost at $2\frac{3}{4}$ cents a pound?

103. If 248 men, in 6 days of 11 hours, dig a trench 232*yd.* long, 8*ft.* wide and 7*ft.* deep, in how many days of 9 hours will 39 men dig a trench 300*yd.* long, 14*ft.* wide and 8*ft.* deep?

104. In how many days, by travelling 10 hours a day, can I accomplish a journey of 1350 miles, if, at the same rate of travel, I can go 475 miles in 4 days, by travelling $7\frac{1}{2}$ hours a day?

105. If 17*yd.* of cloth that is 26*in.* wide, cost \$18.75, what will 36 yards of similar quality cost, that is 32*in.* wide?

106. If 120 flags, each a foot square, will pave a court, how many flags, each 9*in.* long and 6*in.* wide, would be required for the same pavement?

107. How many stones, each 1 6′ long and 9′ broad, would be required to pave an area, which may be covered by 840 stones, each 3*ft.* long and 2*ft.* broad?

108. A load of hay 11*ft.* long, 6*ft.* wide and 8*ft.* high, was sold for \$15.75. What would be the price of a load 10*ft.* long, 4*ft.* wide and 6*ft.* high, at the same rate?

109. If 5 men, with 7 horses each, can haul 315*T.* 11*cwt.* in 3 days of 7 hours, how much will 13 men, with 3 horses each, haul in 9 days of 6 hours?

110. If 2 men, with 8 yokes of oxen each, can haul $170\frac{2}{5}T.$ in 1 day of 11 hours, how many hours a day must 4 men, with 6 yokes each, work, in order to haul $1199\frac{1}{2}T.$ in 5 days?

111. If a cistern $17\frac{1}{2}$*ft.* long, $10\frac{1}{2}$*ft.* broad, and 13*ft.* deep, hold 546 barrels, how many barrels will fill a cistern that is 16*ft.* long, 7*ft.* broad, and 15*ft.* deep?

112. If by selling 15 barrels of flour for \$75.00, I gain 15 per cent., what per cent. shall I gain by selling 31 barrels at \$149?

113. If a pail of water weighs 25 pounds, what would be the weight of a piece of gold of $\frac{4}{5}$ the bulk, the specific gravity of gold being 19.258?

114. A building recently destroyed by fire, was constructed by 45 men in 11 months, by working 9 hours a day. How many men will build one of 4 times the size, in 5 months, working 12 hours a day?

115. A wall which is to be built 18 feet high, was raised 12 feet by 21 men, in 7 days. How many men must be employed to complete it in $2\frac{5}{8}$ days?

116. Since a piece of land 16 rods long and 10 rods wide makes an acre, what must be the length of a field that is 35 rods wide, to contain $27\frac{5}{8}$ acres?

117. If 7 men, in $8\frac{1}{2}$ days, by working 9 hours a day, can build $\frac{1}{8}$ of a wall that is to be raised 12 feet, how many days must 11 men work, when the days are 8 hours long, to raise the same wall 5 feet?

118. How much must I pay for carrying 11 boxes, each of which weighs $3\frac{1}{2}$*cwt.*, 75 miles, if the freight of 8 boxes, each weighing 4*cwt.*, is \$6.50 for 43 miles?

119. If 5 men, by working 8 hours a day for 12 days, can build a wall 40 rods long, 2 feet thick, and 6 feet high, how many men, working 9 hours a day, will build a similar wall, 30 rods long, 3 feet thick and 8 feet high, in 4 days?

120. If a man, by travelling 8 hours a day, for $11\frac{1}{2}$ days, can accomplish a journey of 460 miles, how far would he go in 17 days, by travelling half as fast again for 9 hours a day?

121. Two couriers travel at a uniform rate in opposite directions, for 5 days, riding $11\frac{1}{2}$ hours each day. How far apart are they at the end of the fifth day, if a locomotive, which goes twice as fast, will run 360 miles in 3 days of 8 hours?

122. If 13 horses eat $4\frac{1}{2}$*cwt.* of hay in $3\frac{1}{2}$ days, how much will 78 horses eat in 14 days?

123. A pile of wood 60*ft.* long, 10*ft.* high, and 8*ft.* thick, was sold for \$$238\frac{1}{8}$. What would be the price of a pile 20*ft.* long, 8*ft.* high and 4*ft.* thick, at the same rate?

124. If a pile of wood $31\frac{1}{4}$*ft.* long, $9\frac{1}{4}$*ft.* high and $7\frac{3}{4}$*ft.* thick, cost \$101.50, what must be the length of a pile 13*ft.* high and $6\frac{3}{4}$*ft.* thick, to cost \$$79.87\frac{1}{2}$?

125. How many oxen can be pastured $13\frac{2}{3}$ weeks, for \$$143\frac{1}{2}$, if it cost \$$266\frac{1}{2}$ to pasture 26 oxen $6\frac{5}{8}$ weeks?

126. How many horses will eat 384*qt.* of oats in 16 days, if 17 horses eat 1292*qt.* in 19 days?

127. In how many days will 31 horses eat 58*bu.* 4*qt.* of oats, if 14 horses eat 6*bu.* 2*pk.* 2*qt.* in 3 days?

128. How many men, by working 12 hours a day, will accomplish as much work in 24 days, as 16 men can accomplish in 36 days, by working 10 hours a day?

129. If 2500 slates, each 8 inches long and 5 inches wide, will cover a roof, how many will be required that are 6 inches long and 4 inches wide?

130. If 3 compositors set $15\frac{1}{2}$ pages of type in $13\frac{3}{8}$ hours, how many will be required to set $69\frac{3}{4}$ pages of the same work in $6\frac{1}{4}$ hours?

131. If 12 oxen eat $7\frac{3}{5}$ tons of hay in 22 weeks, how many oxen will eat $13\frac{2}{3}$ tons in $43\frac{3}{19}$ weeks?

132. If 18 men, in $9\frac{1}{2}$ months, consume flour worth $78.75, when wheat is $$1.12\frac{1}{2}$ per bushel, how many months will $145 supply 35 men with flour, when wheat is $1.00 per bushel?

133. If a cellar 22.5*ft.* long, 17.3*ft.* wide, and 10.25*ft.* deep, be dug in 2.5 days by 6 men, working 12.3 hours a day, in how many days of 8.2 hours, would 9 men dig a cellar 56.25*ft.* long, 25.95*ft.* wide, and 16.4*ft.* deep?

134. If 36 men, in $127\frac{1}{2}$ days of $13\frac{1}{2}$ hours, dig a trench $33\frac{3}{4}$*yd.* long, $10\frac{1}{2}$*ft.* deep, and $15\frac{3}{5}$*ft.* wide, how many men, in $7\frac{2}{7}$ days of $12\frac{8}{11}$ hours, will dig a similar trench $82\frac{8}{11}$*yd.* long, $7\frac{2}{5}$*ft.* deep, and 10*ft.* wide?

135. If 19 men, in $71\frac{3}{4}$ days of $10\frac{3}{4}$ hours, dig a trench $41\frac{1}{2}$*yd.* long, $5\frac{7}{8}$*ft.* deep, and $7\frac{4}{5}$*ft.* wide, how long a trench that is $8\frac{2}{5}$*ft.* deep and $4\frac{7}{10}$*ft.* wide, will 11 men dig in $291\frac{1}{2}$ days of $4\frac{7}{9}$ hours?

136. How many days of $8\frac{1}{2}$ hours will 42 men require, to build a wall $98\frac{3}{4}$*ft.* long, $7\frac{1}{2}$*ft.* high, and $2\frac{3}{4}$*ft.* thick, if 63 men can build a wall $45\frac{1}{3}$*ft.* long, $6\frac{7}{12}$*ft.* high, and $3\frac{1}{8}$*ft.* thick, in 68 days of $11\frac{1}{3}$ hours?

137. If 100 pounds at Rochelle equal 106*lb.* at Leipsic, 118*lb.* at Leipsic equal 163*lb.* at Leghorn, 137*lb.* at Leghorn equal 100*lb.* at Seville, 101*lb.* at Seville equal $158\frac{1}{8}$*lb.* at Venice, 140*lb.* at Venice equal 83*lb.* at Franckfort, and $89\frac{1}{2}$*lb.* at Franckfort equal 100*lb.* Avoirdupois, how many *lb.* Avoirdupois equal 350*lb.* at Rochelle?

138. If 750 tiles, each 10*in.* long and 8*in.* wide, will pave an area $12\frac{1}{2}$*ft.* wide, how many tiles that are 16*in.*

long and 9*in.* wide, will pave an area of twice the length and 15*ft.* wide?

139. If \$$328\frac{1}{3}$ support a family of 16 persons $9\frac{4}{7}$ months, each person consuming $4\frac{1}{2}$*lb.* per day, how much would support a family of 28 persons $7\frac{4}{5}$ months, each consuming $5\frac{1}{8}$*lb.* per day?

CHAPTER XII.

FELLOWSHIP.

By the rule of Fellowship, we determine the gain or loss of partners in trade or joint adventures, the worth of debts in bankruptcy, &c.

The whole gain or loss should evidently be distributed among the stockholders, in proportion to each man's share: hence we have the following

RULE.

Divide the whole gain or loss by the whole stock, for the gain or loss of one dollar, and multiply the quotient reduced to its lowest terms, by each man's share.

1. A., B. and C. entered into partnership. A. contributed \$500, B. \$300, and C. \$600. At the end of a year they had gained \$350. What was the share of each?

2. The debts of a bankrupt amount to \$1275, and his whole property is worth but \$950. What proportion of the whole can he pay, and what is the value of a debt of \$361.25?

3. A vessel during a storm sustained a damage of \$1950. How much of the loss should be sustained by A., whose share of the cargo was \$1000, the whole value of the vessel and cargo being \$21500?

4. William Johnson, Samuel Buffum, and Moses Clark, entered into partnership. Johnson contributed \$4750, Buffum \$5250, and Clark \$6000. At the end of three years they had lost \$2500. What was the loss of each?

5. A., B. and C. traded in company; A. put in \$3000, B. \$2000, and C. the remainder. They gained \$1500, of

which C. took $500. What did C. contribute, and what was A.'s share of the gain?

6. The property of a bankrupt is worth but $5153.90, and he owes A. $1765.75, B. $540.00, C. $2329.50, and D. $1380.10. How much will each creditor receive?

7. A. contributed $380 to an adventure, B. $120, C. $500, and D. $700. What was each man's share of the gain, which was $900?

The use of $500 for 3 months, is worth as much as the use of $1500 for 1 month. Therefore, when the stock of the several partners is employed for unequal times, *multiply each stock by the time it is employed, and work with the products as with the original stocks in the general rule.*

8. In a certain firm A. contributes $5000 for 4 months, B. $6000 for 3 months, C. $2000 for 6 months, D. $4000 for 5 months, and E. $5000 for 2 months. How should the gain, which was $1680, be divided among the partners?

9. A. and B. form a limited partnership for 2 years and 6 months. A. at first contributed $2000, and in 13 months $1000 more. B. contributed at first $2500, and at the end of the first year $600 more, but at the end of 19 months he withdrew $2000. At the end of 14 months, C. was admitted to the firm, and contributed $3700. How should the gain, which was $3000, be divided?

10. A. and B. traded in company. A. contributed at first $800, and B. $1000. At the end of 3 months A. advances $1200 more. What sum must B. add to the firm at the end of 6 months, to entitle him to one-half of the year's profits?

11. The amount of the legacies bequeathed in a will was $35000, but the whole amount of the testator's property was found to be only $31500. What was the value of the wife's legacy, which was $10000, and of the eldest son's, which was $15000?

12. A., B. and C. form a joint stock of $1098, by which they gain $234. A.'s money is in trade 4 months, B.'s 5 months, and C.'s 13 months, and their shares of the gain

are as the numbers 2, 3 and 4, respectively. What was each man's stock and gain?

13. A privateer captured a prize valued at \$15000, which the crew agree to divide in proportion to their pay, and the time each has been on board; the officers and midshipmen have been on board 4 months, and the sailors 3 months; the pay of the officers is \$20 a month, of the midshipmen \$15, and of the sailors \$12, and there are 3 officers, 6 midshipmen, and 40 sailors. How much does each receive?

14. Three men hire a pasture for \$40. A. put in 4 horses for 3 months and 6 for 2 months, B. put in 5 for 4 months and 10 for 1 month, and C. put in 7 for 4 months and 3 for 2 months. How much ought each to pay?

15. A man failing in trade owed \$75000, to meet which he had property valued at \$14500. How much can he pay A., who is a creditor for \$10000, B., who is a creditor for \$3750, and C., who is a creditor for \$12362.50?

16. A. commenced business with a capital of \$2500. Five months afterwards he received B. as a partner, who added \$4000 to the stock. Six months after B. was admitted, C. joined the firm with a capital of \$6000. Two years from the commencement, they had gained \$8000. What was each man's share of the gain?

17. It is required to divide \$36000 among 4 persons, in such manner that the second may have twice as much as the first, the third as much as the first and second, and the fourth three times as much as the third?

18. A legacy of \$40000 was left to four heirs, in the proportion of $\frac{1}{6}$ to the first, $\frac{2}{5}$ to the second, $\frac{4}{9}$ to the third, and $\frac{1}{3}$ to the fourth. What was the share of each?

19. A levy of 12[illegible]0 horses is to be distributed among three regiments in the proportion of 1, $\frac{8}{11}$ and $\frac{7}{9}$, respectively. How many will each regiment receive?

20. Five men hire a pasture for \$294. A. put in 10 cows for 3 months; B. put in 5 cows for 2 months; C. put in 8 cows for $1\frac{1}{2}$ months; D. put in 16 cows for $2\frac{1}{4}$ months; and E. put in 20 cows for 4 months. How much ought each to pay?

21. A bankrupt owes $150000, to pay which he has the following property: real estate worth $17500, merchandise worth $3750, personal property worth $20000, and sums due from various individuals amounting to $100000. If he collects all his dues, how much can he pay on every dollar of his debts?

22. A., B. and C. purchase a house for $10000, A. contributing $4000, B. $3600, and C. $2400. The house rents for $800, and the taxes and repairs amount to $50 a year. What income does each of the owners derive from the estate, and what per cent. of his investment?

23. Divide 7500 into 5 parts, in the proportions of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$.

24. In the distribution of a bankrupt's property, $763.50 was divided among four of his creditors. A.'s bill was $250, B's. $300, C.'s $325, and D.'s $362.50. How much did each receive?

25. Four men were joint owners of a farm, their shares being in the proportion of 6, 2, 5, and 3, respectively. It is required to divide the annual rent, which is $1500, equitably among them.

CHAPTER XIII.

ALLIGATION.

When the values of a variety of ingredients are given, and it is desired to make a mixture of any fixed value, the quantity of each ingredient to be employed is determined by ALLIGATION.

When there is no limitation in the quantity of either ingredient, we may make the desired mixture by the following

RULE.

Having written the values of the ingredients in a perpendicular column, connect by a line each value that is less than the required average with one or more that is greater, and each value that is greater with one or more that is less.

Write the difference between each value and the average, op-

posite the ingredient with which that value is connected, and the difference (or the sum of the differences, if there be more than one) opposite each ingredient will be the quantity of that ingredient required.

EXAMPLE FOR THE BOARD.

How much sugar, at 5*cts.*, 7*cts.*, 8*cts.*, 10*cts.*, and 12*cts.*, must be mixed together, that the mixture may be worth 9*cts.* a pound?

$$9\left\{\begin{array}{l}5\\7\\8\\10\\12\end{array}\right.\begin{array}{|l}1+3\\1\\1+3\\4+2+1\\4+1\end{array}\qquad 9\left\{\begin{array}{l}5\\7\\8\\10\\12\end{array}\right.\begin{array}{|l}3\\3\\1\\1\\4+2\end{array}\qquad 9\left\{\begin{array}{l}5\\7\\8\\10\\12\end{array}\right.\begin{array}{|l}1\\3\\3\\4\\2+1\end{array}$$

1st Ans. 4*lb.* at 5, 1 *lb.* at 7, 4*lb.* at 8, 7*lb.* at 10, 5*lb.* at 12. | 2d Ans. 3*lb.* at 5, 3*lb.* at 7, 1*lb.* at 8, 1*lb.* at 10, 6*lb.* at 12. | 3d Ans. 1*lb.* at 5, 3*lb.* at 7, 3*lb.* at 8, 4*lb.* at 10, 3*lb.* at 12.

We may obtain as many answers as there are different ways of connecting the numbers above, with those below the average.

To prove the rule correct, let us examine the second of the above answers. If we were mixing sugars at 5 and 12*cts.* to sell the mixture at 9*cts.*, we should gain 4*cts.* on every pound of the former, and lose 3*cts.* on every pound of the latter. Then, on 3*lb.* of the former we should gain 12*cts.* and on 4*lb.* of the latter we should lose 12*cts.*; therefore, if we mix these quantities, we shall neither gain nor lose by selling the mixture at 9*cts.* In the same way it may be shown that 3*lb.* at 7*cts.* and 2*lb.* at 12*cts.*, 1*lb.* at 8*cts.* and 1*lb.* at 10*cts.* may be sold at the average of 9*cts.*, and the same reasoning will prove the truth of each of the other answers.

1. How much tea at 50*cts.* 75*cts.* 90*cts.* and $1.00, must be taken to form a mixture worth 80*cts.*

2. A jeweller has gold of 16, 17, 18, 20, 22, and 24 carats fine. What proportion of each must he take to make a mixture 21 carats fine? Pure gold is 24 carats fine, or $\frac{24}{24}$.

3. How much grain, at 50*cts.* 75*cts.* $1.00, and $1.10 per bushel, will make a mixture worth 90*cts.* a bushel?

4. How much water must be mixed with wine at $1.50 and $2.00 a gallon, to make the whole worth $1.00 per gallon?

5. What quantity of raisins, at 10*cts.* 18*cts.* and 20*cts.* per *lb.* must be mixed together, to fill a cask containing 150*lb.* and to be worth 19*cts.* a *lb.*? [After obtaining the

proportions by Alligation, find the exact quantities by Fellowship.]

6. It is required to mix sugar at 7*cts.* 8*cts.* 10*cts.* and 12*cts.* per *lb.* in such manner as to form a mixture of 2*cwt.* 3*qr.*, worth 11*cts.* per *lb.*

7. Mix tobacco at 8*cts.* 10*cts.* 12*cts.* and 16*cts.* so as to make 100 pounds worth 11*cts.* a pound.

EXAMPLE FOR THE BOARD.

A farmer wishes to mix 10 bushels of barley at 50*cts.*, 4 bushels of oats at 45*cts.*, and 16 bushels of rye at 75*cts.* with wheat at $1.25 and corn at 90*cts.* a bushel, so that the mixture may be worth $1.00 per bushel.

We may regard the limited quantities as a single ingredient of 30 bushels, worth 62*cts.* a bushel. Proceeding in the usual way,

$$1.00\left\{\begin{array}{l|l} 62 & 25 \\ 90 & 25 \\ 1.25 & 38+10 \end{array}\right.$$

we find that 25 bushels at 62*cts.*, 25 at 90*cts.*, and 48 at $1.25, would give us a mixture of the desired average value. But as we have 30 bushels at 62*cts.*, we must take $\frac{30}{25}$ or $\frac{6}{5}$ of these proportionate quantities, and we have 30 bushels at 90*cts.* and $57\frac{3}{5}$*bu.* at $1.25, for the Answer.

8. How much water must be mixed with 40 gallons of syrup, at 50*cts.* a gallon, to make the whole worth $37\frac{1}{2}$*cts.* a gallon?

9. A grocer has 10 gallons of wine at 75*cts.*, 12*gal.* at 90*cts.*, and 8*gal.* at $1.00, with which he would mix brandy at $1.25, and water, so as to make a mixture worth 95*cts.* a gallon. How much of each must he take?

10. If a cubic inch of gold weighs 11.11*oz.* and a cubic inch of silver 6.04*oz.*, what quantity is contained in a mass of 63*oz.*, of which 1 cubic inch weighs 8.35*oz.*?

11. How much molasses, at 50 cents, and water, must be mixed with 15 gallons at $37\frac{1}{2}$ cents, 28 gallons at 25 cents, and 19 gallons at 33 cents, to make a mixture worth 31 cents a gallon?

12. A grocer has an order for 150*lb.* of tea, at 90 cents per *lb.*, but having none at that price, he would mix some at 75 cents, some at $87\frac{1}{2}$ cents, and some at $1.00 per pound. How much of each sort must he take?

CHAPTER XIV.

PERMUTATION AND COMBINATION.

PERMUTATION shows the number of changes that can be made in the order of a given number of things.

PROBLEM I.

To find the number of changes that can be made of any given number of things, all different from each other.

How many changes may be made in the position of 4 persons at table?

If there were but two persons, a and b, they could sit in but two positions, ab and ba. If there were three, the third could sit at the head, in the middle, or at the foot, in each of the two changes, and there could then be $1\times2\times3=6$ changes. If there were 4, the fourth could sit as the 1st, 2d, 3d, or 4th, in each of these 6 changes, and there would then be $1\times2\times3\times4=24$ changes.

RULE.

Multiply together the series of numbers 1, 2, 3, &c., up to the given number, and the product will be the number sought.

1. How many variations may be made in the order of the 9 digits?

2. How many changes may be made in the position of the letters of the alphabet?

3. How long a time will be required for 8 persons to seat themselves at table in every possible order, if they eat 3 meals a day?

PROBLEM II.

Any number of different things being given, to find how many changes can be made out of them by taking a given number of the things at a time.

If we have five things, each one of the 5 may be placed before each of the others, and we thus have 5×4 permutations of 2 out of 5. If we take 3 at a time, the third thing may be placed as 1st, 2d, and 3d, in each of these permutations, and we have $5\times4\times3$ permutations of 3 out of 5. For a similar reason we have $5\times4\times3\times2$ permutations of 4 out of 5, &c.

RULE.

Take a series of numbers, commencing with the number of things given, and decreasing by 1, until the number of terms is

equal to the number of things to be taken at a time: the product of all the terms will be the answer required.

4. How many changes can be rung with 8 bells, taking 5 at a time?

5. How many numbers of 4 different figures each, can be expressed by the 9 digits?

6. In how many different ways may 10 letters of the alphabet be arranged?

PROBLEM III.

To find the number of permutations in any given number of things, among which there are several of a kind.

How many permutations can be made of the letters in the word *terrier*?

If the letters were all different, the permutations, according to Problem I. would be $1\times2\times3\times4\times5\times6\times7=5040$. But the permutations of the three r's would, if they were all different, be $1\times2\times3$, which could be combined with each of the other changes; the number must therefore be divided by $1\times2\times3$. For the same reason it must also be divided by 1×2, on account of the 2 e's. Then the true number sought is $\frac{1\times2\times3\times4\times5\times6\times7}{1\times2\times3\times1\times2}=420$.

RULE.

Take the natural series, from 1 up to the number of things of the first kind, and the same series up to the number of things of each succeeding kind, and form the continued product of all the series.

By the continued product divide the number of permutations of which the given things would be capable if they were all different, and the quotient will be the number sought.

7. How many changes can be made in the order of the letters in the word Philadelphia?

8. How many different numbers can be made, that will employ all the figures in the number 11908907343?

9. How many permutations can be made with the letters in the word Cincinnati?

COMBINATION shows in how many ways a less number of things may be chosen from a greater.

If we have ten articles, each may be combined with every one of the nine remaining ones, and therefore we may have 10×9 permutations of 2 out of 10. But each combination will evi-

dently be repeated; thus, we shall have *ab* and *ba*, *ac* and *ca*, &c. Therefore, the number of combinations will be $\frac{10\times 9}{2}$.

If now we add an eleventh article, each of the eleven may be joined to each of the combinations of the remaining ten, and we shall have $\frac{11\times 10\times 9}{1\times 2}$ permutations. But each combination will be three times repeated; thus we shall have *abc*, *bac*, and *cab*; *abd*, *bad*, and *dab*, &c. The number of combinations of 3 out of 11 will therefore be $\frac{11\times 10\times 9}{1\times 2\times 3}$. Hence we obtain the following

RULE.

Write for a numerator the descending series, commencing with the number from which the combinations are to be made, and for a denominator the ascending series, commencing with 1, giving to each series as many terms as are equivalent to the number in one combination.

Cancel the like factors in the numerator and denominator, and divide.

10. How many combinations of 4 letters, can be made from the alphabet?

11. How many combinations of 7 can be made from 18 apples?

12. How many ranks of 10 men, may be made in a company of 80?

13. How many locks of different wards, may be unlocked with a key of 6 wards? [Find the number of combinations of 1, 2, 3, 4, 5, and 6 in 6, and the sum of all the combinations will be the number required.]

CHAPTER XV.

INVOLUTION.

INVOLUTION is the repeated multiplication of a number by itself.

The product obtained by Involution is called a *power*. The *root* is the number involved, or the *first power*. If the root be multiplied by itself, or employed twice as a factor, the product is the second power. If the root is

employed three times as a factor, it is raised to the 3d power; if 5 times, to the 5th power, &c. Thus, 2 is the 1st power of 2, or 2^1. 2×2 or 4, is the 2d power of 2, or 2^2. $2\times2\times2$ or 8, is the 3d power of 2, or 2^3. $2\times2\times2\times2\times2$ or 32, is the 5th power of 2, or 2^5. The power is usually denoted by a small figure over the right of the root, called the *exponent*, or *index*. When there is no exponent, the number is regarded as the 1st power.

The second power is often called the square, because the number of square feet in any square surface, is obtained by multiplying the number of feet in one side by itself.

The third power is often called the cube, because the number of cubic feet in any cubical block, may be obtained by raising the number of feet in one side to the 3d power.

The 4th power is sometimes called the bi-quadrate, or the square squared; the 5th power, the first sursolid; the 6th power, the square cubed, or the cube squared; the 7th power, the second sursolid; the 8th power, the bi-quadrate squared; the 9th power, the cube cubed; the 10th power, the 1st sursolid squared, &c.

If the exponents of any two powers of the same number be added, we shall obtain the exponent of their product. Thus $6^3\times6^5=6\times6\times6\times6\times6\times6\times6\times6=6^8$; $4^2\times4^3=4\times4\times4\times4\times4=4^5$.

In any two powers of the same number, if we subtract the smaller exponent from the larger, we shall obtain their quotient. Thus $6^8\div6^5=\frac{6\times6\times6\times6\times6\times6\times6\times6}{6\times6\times6\times6\times6}=$ $6\times6\times6=6^3$.

We may represent any power of a number, by multiplying its exponent. Thus, the 7th power of 5 is 5^7; the 3d power of 2^2 is 2^6, because $2^2\times2^2\times2^2=2^6$. These properties form the basis of the system of Logarithms.

1. What is the 2d power of 6? the 3d power?
2. Find the value of $.9^4$; 12^3; $(\frac{1}{2})^5$; 2^9.
3. Find the value of 16^4; 1.6^4; $.16^4$; $(\frac{1}{13})^3$.
4. What is the square of 13.68? of $9\frac{2}{5}$?

5. What is the difference between 3^4 and 4^3?

6. What is the value of 1^{17}; 3^7; $2^3 \times 2^2$?

7. What power of 9 is equivalent to $9^5 \times 9^3$; $9^2 \times 9^{10}$; $9^4 \times 9^6$; $9 \times 9^7 \times 9^8$?

8. Multiply 127^9 by 127^7, and divide the product by 127^{15}.

9. Divide 31^9 by 31^9; 17^8 by 17^5; 42^7 by 42^6.

10. What is the sixth power of $4\frac{1}{10}$?

11. What is the 9th power of 5^3? the 12th power of 18^5? the 24th power of 17^2?

CHAPTER XVI.

EVOLUTION.

Evolution is the process by which we discover the root of any given power. Thus, 3 is the 2d root of 9, the 3d root of 27, the 5th root of 243, because $9=3^2$, $27=3^3$, $243=3^5$. So the 2d or square root of 49 is 7; the 3d or cube root of 125 is 5; the 4th root of 16 is 2; the 5th root of 1024 is 4, &c. We may denote a root by a ***radical sign***, or by a ***fractional exponent***.

The radical sign is $\sqrt{}$, and when employed by itself denotes the square root. If we wish to denote the 3d, 5th, 7th, &c. root, the index of the root is written above the radical sign thus, $\sqrt[3]{}$, $\sqrt[5]{}$, &c. In fractional exponents, the numerator expresses the power of the number, and the denominator expresses the root. Thus, $(27)^{\frac{1}{3}} = \sqrt[3]{27}$; $(16)^{\frac{3}{4}} = \sqrt[4]{16^3}$; $(32)^{\frac{4}{5}} = \sqrt[5]{32^4}$, &c.

The product, or the quotient, of two second, third, or other roots, is the 2d, 3d, &c., root of the product or quotient. Thus, $\sqrt[3]{27} \times \sqrt[3]{125} = \sqrt[3]{27 \times 125}$ or $\sqrt[3]{3375}$. For $27=3^3=3\times3\times3$, and $125=5\times5\times5$. Then $27\times125= 3\times3\times3\times5\times5\times5=3\times5\times3\times5\times3\times5=15^3$. Therefore $\sqrt[3]{27\times125}=15$. In a similar manner it may be shown that $\sqrt[3]{3375} \div \sqrt[3]{125} = \sqrt[3]{27}$.

The power of any root may be obtained by multiplying the fractional exponent. Thus the 4th power of $27^{\frac{2}{3}}=27^{\frac{8}{3}}$. For by the last proposition $\sqrt[3]{27^2}\times\sqrt[3]{27^2}\times\sqrt[3]{27^2}\times\sqrt[3]{27^2}=\sqrt[3]{27^8}=27^{\frac{8}{3}}$.

The root of any power or root may be obtained by dividing the exponent by the index of the desired root. Thus $\sqrt[3]{3^{\frac{1}{5}}}=3^{\frac{1}{5}\div 3}=3^{\frac{1}{15}}$.

This is the converse of the last proposition. For if the 3d power of $3^{\frac{1}{15}}$ is $3^{\frac{3}{15}}$, or $3^{\frac{1}{5}}$, the 3d root of $3^{\frac{1}{5}}$ must be $3^{\frac{1}{15}}$.

If the numerator and denominator of fractional indices be multiplied or divided by the same number, the value of the quantity is not altered. Thus, $3^{\frac{4}{6}}=3^{\frac{8}{12}}=3^{\frac{2}{3}}$. For the multiplication of the numerator involves the number to a certain power, and the multiplication of the denominator extracts the corresponding root. Then the 3d root of the 3d power, the 5th root of the 5th power, &c., is the 1st power.

We may multiply or divide any two roots of the same number, by adding or subtracting the fractional exponents. Thus, $\sqrt[3]{2}\times\sqrt{2}=2^{\frac{1}{3}+\frac{1}{2}}=2^{\frac{5}{6}}$; $\sqrt[3]{5}\div\sqrt[4]{5}=5^{\frac{1}{3}-\frac{1}{4}}=5^{\frac{1}{12}}$. For by the last proposition we have $\sqrt[3]{2}\times\sqrt{2}=\sqrt[6]{2^2}\times\sqrt[6]{2^3}$, which is equivalent to $\sqrt[6]{2^5}$ or $2^{\frac{5}{6}}$. Also $\sqrt[3]{5}\div\sqrt[4]{5}$, $\sqrt[12]{5^4}\div\sqrt[12]{5^3}=\sqrt[12]{5}$ or $5^{\frac{1}{12}}$.

When the exact root of a number can be obtained, it is called a *rational number*. An *irrational number*, or *surd*, is one whose exact root cannot be obtained. Thus, $\sqrt{16}$, $\sqrt[3]{27}$, $\sqrt[3]{64}$, $\sqrt[4]{81}$, are rational numbers, equivalent to 4, 3, 4, 3, respectively. But $\sqrt{5}$, $\sqrt[3]{19}$, $\sqrt[5]{16}$, are all surds, and their roots can only be obtained approximately.

A number which has a rational root, is called a perfect power. Thus, 16 is a perfect 2d power, and a perfect 4th power, but an imperfect power of any other degree. But 5, 7, 12, &c., are imperfect powers of any degree.

1. What is the square root of 9? the cube root of 8?

2. What is the 4th root of 81? the 5th root of 32?

3. What is the value of $\sqrt[5]{1}$; $25^{\frac{1}{2}}$; $\sqrt[3]{64}$; $64^{\frac{8}{16}}$?

4. What is the product of $\sqrt[4]{8}$ by $\sqrt[4]{12}$; $\sqrt[7]{9}$ by $7^{\frac{1}{7}}$?

5. Multiply $\sqrt[4]{3}$ by $\sqrt[6]{3}$; $7^{\frac{1}{2}}$ by $\sqrt[9]{7}$; $4^{\frac{2}{3}}$ by $\sqrt[4]{4^2}$.

6. Divide $6^{\frac{1}{3}}$ by $6^{\frac{1}{5}}$; $\sqrt[4]{5}$ by $\sqrt[5]{5}$; $17^{\frac{1}{3}}$ by $\sqrt[9]{17}$.

7. Find the 4th power of $\sqrt{9}$; the 6th power of $8^{\frac{4}{5}}$.

8. What is the cube root of 7^6; the 5th root of $11^{\frac{4}{7}}$?

EXTRACTION OF THE SQUARE ROOT.

The square root of a number is *the number which, when multiplied by itself, will produce the given number.*

In the following table are the numbers from 1 to 10 inclusive, and beneath them are their squares; therefore, the numbers of the second line have for their square roots the numbers of the first.

Roots	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Squares	1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

If there are decimals in the root, there will be twice as many in the square: because any product contains as many decimals as both factors. And conversely, there will be half as many decimals in the root as in the square.

Every entire number, which is not the square of another entire number, is an imperfect second power. For the root of such a number cannot be expressed by a fraction, because a fraction multiplied by itself would give a fractional product,—it must therefore be a surd.

The least square of units, is.............	$1\times1 = 1$
The least square of tens is..............	$10\times10 = 100$
The least square of hundreds, is.........	$100\times100 = 10000$
The greatest square of units, is..........	$9\times9 = 81$
The greatest square of tens and units, is....	$99\times99 = 9801$
The greatest square of hundreds, tens and units, is.........................	$999\times999 = 998001$

Hence we may see that if we divide a square number into periods of two figures, by placing a point over units, and one over each second figure to the left, the number of periods will denote the number of figures in the root.

Thus, the square root of $\dot{1}4\dot{4}$ contains two figures, and is 12. The square root of $\dot{1}6\dot{0}0$ contains 2 figures, and is 40.

The square of 30 is 900. The square of 37 is 1369, and to discover in what manner this square is formed, we will multiply 37 by 37, writing each product separately, instead of adding them as we proceed. We then see that we must add the square of the tens, twice the product of the tens by the units, and the square of the units.

$$\begin{array}{r} 37 \\ 37 \\ \hline 49 \\ 21 \\ 21 \\ 9 \\ \hline 1369 \end{array}$$

Let us now reverse the process, and extract the square root of 1369. Pointing the periods, we find the root will consist of two figures, and the square of the tens must therefore be contained in the 13 hundreds. The greatest square in 13 is 9, and the root 3 is written as the first figure of the required root. Subtracting the square of the root already found, the remainder, 469, must contain twice the product of the tens by the units + the square of the units. To obtain the units' figure, we divide the 46 tens by $2\times3=6$ tens, which gives a quotient 7. Writing the 7 in the root, and also at the right of the divisor, we multiply by 7, and obtain 469, which is *twice the product of the tens by the units plus the square of the units.* Hence we deduce the following

$$\begin{array}{rl} & \dot{1}3\dot{6}9\,(\,37 \\ & 9 \\ \hline 67) & 469 \\ & 469 \\ \hline \end{array}$$

RULE.

Separate the number into periods of two figures each, by placing a point over the units' figure, and another over each second figure to the left (and also to the right, if decimals are desired in the root). Write in the quotient the root of the greatest square contained in the left hand period, and subtract its square from the period.

To the remainder annex the second period, and divide the tens of the number thus formed, by twice the first quotient figure, placing the result in the root, and also at the right of the new divisor. Multiply the completed divisor by the new quotient figure, subtract the product from the dividend, and annex the third period to the remainder.

Double the root figures already found for a new trial divisor, and proceed as before, until all the periods are brought down.

When any trial divisor is not contained in the tens of

the dividend, place a zero in the root, and also at the right of the divisor, and bring down the next period.

If any figure obtained for the root proves too large, diminish it by one and repeat the work.

Approximate roots may be obtained by annexing decimal periods of two figures each.

1. Extract the square root of 529; of 961.

2. Extract the square root of 1444.

3. What is the square root of 3249? of 3969?

4. What is the square root of 12321?

5. What is the square root of 2256.25?

6. What is the square root of 1.907161?

7. What is the square root of 53300033424?

8. Find the square root of .0361; of .000040804.

9. Find the square root of $\frac{9}{16}$; $\frac{25}{49}$; $1\frac{7}{9}$; $7\frac{29}{100}$.

10. Find the approximate square root of 2, to six decimal places.

11. Find the approximate square root of 63; 1.6; .009; 27.1; 14367.

12. Find the square root of $318\frac{82}{169}$; $7\frac{1}{9}$; $7\frac{9}{16}$; $4\frac{2}{5}$; $17\frac{1}{2}$.

13. How long is the side of a square field that contains 225 acres?

14. What is the side of a square which contains as much as a circle whose area is 273.5 square feet?

15. An army of 8649 men is arranged in a solid square. How many are there on each side?

16. A certain oblong field contains 40 acres, and the length is 4 times the width. Required the length and breadth.

The areas of circles are proportioned to the squares of their diameters.

The areas of any similar figures are proportioned to the squares of their like dimensions.

The area of any circle is equal to the square of its diameter multiplied by .7854.

Fmer

The circumference of a circle is equal to its diameter multiplied by 3.1416.*

The area of a triangle is equal to the base multiplied by half the height.

In any right-angled triangle, the square of the longest side is equal to the sum of the squares of the other two sides.

The distance through which bodies fall, when falling freely, are as the squares of the times. In a vacuum, a body would fall $16\frac{1}{12}$ *ft.* in 1 second. Then we have the proportion, letting n represent any number of seconds.

sec. *sec.* *ft.* *ft.*
$(1)^2 : n^2 :: 16\frac{1}{12}$: distance in n seconds.

Any three terms of this proportion being given, the fourth may be readily found. But it should be remarked, that in consequence of the resistance of the air, the space actually fallen through is somewhat less than that given by the formula.

17. What is the diameter of a circle that is 16 times as large as one whose diameter is 13 feet.

18. The area of a circle is 7632 feet; what is the diameter?

19. A horse is fastened to a post in the centre of a field. What is the length of a rope that will allow him to graze an acre?

20. The base of a triangle is 47 feet. What is the height, the area being 7691 square feet?

21. A ladder 75 feet long, rests against the limb of a tree that is 50 feet from the ground. How far is the foot of the ladder from the root of the tree?

22. The length of a room is 18 feet, and the width 12 feet. What is the distance between the opposite corners? What length of rope would reach from an upper corner to the opposite lower corner, the height being 10 feet?

23. The circumference of a circle is 29 rods. What is the side of a square having an equal area?

24. Two ships left the same port; one sailed 125 miles

* The more exact ratio is, 3.14159265358979323846264338328.

north, the other 100 miles east. How far were they then apart?

25. A kite accidentally lodged in the top of a tree, but the line breaking, I measure its length, which is 210 feet. What is the height of the tree, the base being 189 feet from my standing place?

26. Desiring to know the height of a precipice, I drop a stone from the summit, and observe by my watch that it strikes the ground in $3\frac{1}{2}$ seconds. What is the height?

27. A bag of sand is dropped from a balloon $1\frac{1}{4}$ miles above the surface of the earth. How long will it be in falling?

28. In what time will a stone fall to the bottom of a shaft, that is sunk 870*ft.* below the surface of the ground?

When one number bears the same ratio to a second as the second does to a third, the second number is called a *mean proportional* between the other two. Thus, in the proportion 3 : 6 : : 6 : 12, 6 is a mean proportional between 3 and 12.

The mean proportional between any two numbers is equal to the square root of their product.

29. Find a mean proportional between 7 and 252.

30. Find a mean proportional between .75 and 12.

31. Find a mean proportional between $\frac{1}{5}$ and $\frac{1}{405}$.

32. Find a mean proportional between $\frac{7}{200}$ and .875.

33. Find mean proportionals between $\frac{1}{16}$ and 16; 5 and 6; 25 and 13; $\frac{2}{7}$ and $\frac{5}{9}$.

We may often discover, by simple inspection, that any given number is not a perfect square. The following are some of the principal properties of squares:

(1.) Every even square is divisible by 4. Therefore, no even number, which is not divisible by 4, can be a perfect square.

(2.) If a square number contains a prime factor, it also contains the square of that factor. Therefore, no number divisible by a prime factor, and not divisible by its square, can be a square number.

(3.) All squares terminate in 0, 1, 4, 5, 6, or 9. Therefore, no number terminated by 2, 3, 7, or 8, can be a square number.

(4.) Every square number terminated by 5, is also terminated

by 25. Therefore, no number ending in 5 can be a square, unless the 5 be preceded by a 2. We may remark moreover, that the 2 must be preceded by 0, 2, or 6.

(5.) The zeroes terminating any perfect square, must be of an even number. Therefore, no number terminating in an odd number of zeroes, can be a square number. And if the zeroes be even, unless they are preceded by a square number, the number itself is not a square. Thus 2500 is a square number, but 1500 is not.

EXTRACTION OF THE CUBE ROOT.

The cube root of a number is the *number which, when raised to the third power, will produce the given number.*

In the following table are the numbers from 1 to 10 inclusive, and beneath them are their cubes, therefore the numbers of the second line have for their cube roots the numbers of the first.

Roots	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Cubes	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

Thus we see that there are only nine integral cubes between 1 and 1000. All the other intermediate integers are imperfect cubes, and their roots can only be obtained approximately.

All perfect cubes from 1 to 1000, evidently have but one integral figure in their cube root. All numbers between 1000 or 10^3, and 1000000 or 100^3, will have two figures in their root. And generally, if we divide a cube into periods of three figures each, by placing a point over units, and one over every third figure from units, the number of points will show the number of figures in the root.

EXAMPLES FOR THE BOARD.

In order properly to understand the principles of the cube root, the student should be provided with the following blocks:

1. A cubical block, each side measuring 3 inches, to represent the CUBE OF THE TENS.

2. Three blocks, each 3 inches square and $\frac{7}{10}$ of an inch thick, to represent THE SQUARE OF THE TENS MULTIPLIED BY THE UNITS.

3. Three blocks, each $\frac{7}{10}$ of an inch square and 3 inches long, to represent THE SQUARE OF THE UNITS MULTIPLIED BY THE TENS.

4. A cubical block, each side measuring $\frac{7}{10}$ of an inch, to represent THE CUBE OF THE UNITS.

If block No. 1 be placed upon a table, the three blocks No. 2, laid against three of its sides, the three blocks No. 3, in the deficiencies left by No. 2, and No. 4 in the corner still unfilled, we shall have a new cube, which will represent THE CUBE OF THE TENS, *plus* THREE TIMES THE SQUARE OF THE TENS MULTIPLIED BY THE UNITS, *plus* THREE TIMES THE SQUARE OF THE UNITS MULTIPLIED BY THE TENS, *plus* THE CUBE OF THE ~~TENS.~~ units

This set of blocks is readily applicable in showing the difference between 30^3 and 37^3. But it will be easily perceived that the same result would have been obtained, if, instead of 3 inches, and $\frac{7}{10}$ of an inch, we had employed 4 inches, and $\frac{9}{10}$ of an inch, —11 inches, and $\frac{2}{10}$ of an inch,—or any other numbers to represent tens and units. Therefore, the cube of any number whatever is equal to *the cube of the tens* $+$ *three times the square of the tens* $\times$ *the units,* $+$ *three times the tens* $\times$ *the square of the units,* $+$ *the cube of the units.*

We will now cube 37, both by our rule and by direct multiplication.

The cube of the tens,	$=27$	thous.	37
$3\times$ the square of the tens $\times$ the units,	$=189$	hund.	37
$3\times$ the tens $\times$ the square of the units,	$=441$	tens,	$37^2= 1369$
The cube of the units,	$=343$	units,	37
The cube of 37,	$=50653$		$37^3=50653$

Let it now be required to extract the cube root of 50653.

		50653(37
		27
	tr. div.	
$3\times 3^2 =$	27,49	23653
$3\times 30\times 7=$	6 30	23653
comp. div.	3379	

Dividing the number into periods, we find there will be two figures in the root. 27 is the greatest cube contained in the first period, and its root 3, is evidently the tens' figure of the root sought. Subtracting 27 from 50, and annexing the next period to the remainder, we have 23653, which must evidently contain $3\times$ the square of the tens $\times$ the units, $+3\times$ the tens $\times$ the square of the units $+$ the cube of the units, which is the same as, the units $\times$ ($3\times$ the square of the tens $+3\times$ the tens $\times$ the units $+$ the square of the units). To discover the units' figure, we will employ $3\times$ the square of the tens, or 27 hundreds, for a trial divisor. 27 hundreds is contained in 236 hundreds 8 times. But if we complete our divisor with 8, it will be found too large; we therefore try 7. $3\times$ the tens $\times$ the units $=63$ tens or 630. The square of the units is 49 units, which is annexed to the 27 hundreds. Adding these numbers, we obtain 3379 for our complete divisor, which being multiplied by 7, gives

23653,—the remaining part of the root. Hence we derive the following

RULE.

Separate the number into periods of three figures, by placing a point over the units' figure, and one over each third figure to the left, (and also to the right, if decimals are desired in the root). Write in the quotient the root of the greatest cube contained in the left hand period, and subtract the cube from the period, annexing the second period to the remainder.

Take three times the square of the root already found, for a trial divisor, and find how many times this divisor is contained in the hundreds of the dividend. Place the result in the quotient, and its square at the right of the trial divisor (supplying the place of tens with a zero, if the square is less than ten).

Multiply 30 times the root already found, by the last root figure; this product added to the divisor, will give the complete divisor.

Multiply the complete divisor by the last root figure, subtract the product from the dividend, annex the next period to the remainder, and proceed as before until the whole root is extracted.

If any complete divisor is not contained in its dividend, a zero must be annexed to the root, and two to the trial divisor, and the next period brought down for a new dividend.

If any figure obtained for the root is too great, diminish it by 1 or more, and repeat the work.

[It may be well to illustrate to the pupil the algebraical formation of the cube, by raising $30+7$ to the third power.

In forming the complete divisor, we say, "Multiply 30 times the root already formed, by the last root figure," because $3\times$ the tens$\times$ the units would give tens, and a zero must be annexed for units.

The square of the units is written at the right of the trial divisor, to render the work more compact. This square being units, and the trial divisor hundreds, the two may very properly be united.]

What is the cube root of 226802438.843904?

	226802438.843904(609.84
	$6^3=216$
tr. div.	
$3\times 6^2=108{,}01$	108,02
$30\times 6\times 1=\quad 1\,80$	
10981 c. div.	
which is not contained in the dividend.	
tr. div.	
$30\times 60^2=10800{,}81$	108024,38
$3\times 60\times 9=\quad 16200$	
1096281 com. div.	9866529
tr. div.	
$3\times 609^2=\quad 1112643{,}64$	9359098,43
$30\times 609\times 8=\quad 146160$	
111410524 com.div.	891284192
tr. div.	
$3\times 6098^2=\quad 111556812{,}16$	446256519,04
$30\times 6098\times 4=\quad 731760$	
11156412976 com. div.	44625651904

The trial divisors, after the first, may be more conveniently found, *by adding to the last complete divisor the last number used to complete it, and twice the square of the last root figure.* Thus in the foregoing example, our third trial divisor=

$$\begin{array}{rr} & 1096281 \\ & 16200 \\ 2\times 9^2= & 162 \\ \hline \text{trial divisor,} & 1112643 \end{array}$$

1. What is the cube root of 328509?
2. What is the cube root of 1986091?
3. What is the cube root of 9129329?
4. What is the cube root of 345.357380096?
5. What is the cube root of .062570773?
6. What is the cube root of $\frac{8}{27}$; $\frac{125}{729}$; $\frac{512}{1000}$?
7. What is the cube root of $5\frac{13}{343}$; $11\frac{25}{64}$; $274\frac{5}{8}$?
8. What is the cube root of 6; $\frac{1}{2}$; $2\frac{1}{3}$; 95; 8.71?

9. What is the cube root of $\frac{148877}{1030301}$?

10. What is the cube root of .0081?

11. What is the cube root of 4.160075243787?

The solid contents of similar bodies are to each other as the cubes of their diameters, or of their similar sides.

The solid contents of a sphere may be found by multiplying the cube of the diameter by .5236.

12. What are the solid contents of the earth, supposing it a perfect sphere, whose diameter is 7920 miles?

13. If a ball 2 inches in diameter, weighs $1\frac{1}{8}$ pounds, what would be the weight of a similar ball 6 inches in diameter?

14. What is the side of a cubical box that will hold 1 bushel?

15. What is the side of a cubical pile that contains 258 cords of wood?

16. If a tree 1 foot in diameter, yields 2 cords of wood, how much wood is there in a similar tree that is 3*ft.* 6*in.* in diameter?

17. If a pound avoirdupois of gold is worth $200, and a cubic inch weighs $11\frac{1}{4}$*oz.*, what would be the value of a gold ball 1 foot in diameter?

18. What is the size of a ball that weighs 27 times as much as one 3 feet 6 inches in diameter?

19. If a hollow sphere 3 feet in diameter and $2\frac{1}{3}$ inches thick, weigh 12 tons, what would be the dimensions of a similar sphere that would weigh 324 tons?

20. What is the side of a cubical block of wood that weighs as much as a sphere 15 inches in diameter?

PROPERTIES OF CUBES.

If a cube be divisible by 6, its root will also be divisible by 6. And if a cube, when divided by 6, has any remainder, its root divided by 6 will have the same remainder.

All exact cubes are divisible by 4, or can be made so by adding or subtracting 1.

All exact cubes are divisible by 7, or can be made so by adding or subtracting 1.

All exact cubes are divisible by 9, or can be made so by adding or subtracting 1.

Every cube is divisible by the cube of each of its prime factors.

ROOTS OF HIGHER POWERS.

When the exponent of a power can be resolved into two or more factors, by successively extracting the roots denoted by those factors, we may obtain the root desired. Thus, as $12=3\times2\times2$, the cube root of the square root of the square root of a number, is equal to the 12th root. So the square root of the square root=the 4th root; the cube root of the cube root is the 9th root; the cube root of the square root is the 6th root, &c.

The demonstration of the following rule depends upon Algebraical principles, and therefore cannot properly be introduced here. In its application to square and cube roots, the student will be able to trace some analogy to the rules already given.

GENERAL RULE

FOR EXTRACTING THE ROOTS OF ALL POWERS.

At the left of the number whose root is required, arrange as many columns as are equal to the index of the root, writing 1 at the head of the first or left hand column, and zero at the head of each of the others.

Divide the number into periods of as many figures as the index of the root requires. Write the root of the left hand period as the first figure of the true root.

Multiply the number in the first column by the root figure, and add the product to the second column; add the product of this sum by the root figure to the third column, and so proceed, *subtracting* the product of the last column from the given number.

Repeat this process, stopping at the last column, and thus proceed, stopping one column sooner each time, until the last sum falls in the second column.

To determine the second root figure, consider the number in the last column as a trial divisor, and proceed with the second root figure thus obtained, precisely as with the first.

Continue the operation until the root is completed, or the approximation carried as far as is desired.

In order to avoid error, observe carefully the value of each root figure and each product. Thus, if the first root figure is hundreds, the number in the second column will be hundreds,—in the third, ten thousands,—in the fourth, millions, &c.

EXAMPLES FOR THE BOARD.

What is the third root of 205692449327?

1	0		0	205692449327(5903
	5 thous.		25 mill.	125 bill.
	5		25 c. d.	806,92
	5		50	80379 mill.
	10	(1)	75 t. d.	3134,49327
	5		1431 ten thous.	313449327 un.
(1)	159 hun.		8931 c. d.	
	9		1512	
	168	(2)	10443 t. d.	
	9		53109 un.	
(2)	17703 un.		104483109 c. d.	

The complete divisors are marked c. d., the trial divisors, t. d. The figures at which the new additions commence, are marked (1), (2). The partial dividends by which each root figure is determined, are distinguished by a comma. They always terminate with the first figure of the period that is annexed. The abbreviations, thous., mill., &c., show the value of the figures against which they are placed.

[The extraction of the square root, and the solution of equations of the second power (of which examples are given on p. 125), afford very ready and convenient applications of this rule. The determination of the first root figure in the higher powers, would frequently be difficult, without the aid of Table V. In the solution of many Algebraical Equations, even this table affords no assistance, but we are obliged to rely upon trial, for the first figure of the root, which being ascertained, the succeeding figures will be easily found.]

Extract the 5th root of 858533232.56832.

1	0	0	0	0	858533232.56832(61.2
	6 tens.	36 hund.	216 thous.	1296 ten thou.	7776 hund. thous.
	6	36	216	1296 c. d	8093,3232
	6	72	648	5184	66996301
	12	108	864	(1) 6480 t. d.	139369315,6832
	6	108	1296	2196301 un.	1393693156832
	18	216	(1) 2160	66996301 c. d.	
	6	144	86301 un.	2232904	
	24	(1) 360	2196301	(2) 69229205 t. d.	
	6	301 un.	36603	4554528416 ten thous.	
	(1) 301 un.	36301	2232904	696846578416 c. d.	
	1	302	36906		
	302	36603	(2) 2269810		
	1	303	7454208 thous.		
	303	36906	2277264208		
	1	304			
	304	(2) 37210			
	1	6104 hund.			
	(2) 3052 tenths	3727104			

The additions to the left hand column may be made mentally, and thus shorten the labor. There are other abbreviations, for which the student is referred to the Chapter on Numerical Approximations.

The first root figure in each of the following examples, may be found by the table of Powers and Roots.

1. Extract the square root of 350026681.
2. Extract the square root of 3; 5; 6.5.
3. Extract the cube root of 2924207.
4. Extract the cube root of 13; 12.5.
5. Extract the fifth root of 65.7748550151.
6. Extract the 7th root of 1.2466882923536245063 68.

To show the universal application of the rule, we will solve a question containing many of the powers of a number. At the head of the columns we write the coefficients* of all the powers, from the highest to the lowest, substituting a 0 when any power is wanting. The first coefficient is then multiplied by the root figure, and the product ad-

* A coefficient is a figure indicating the number of times any term is employed. Thus, in 7 times the 5th power, 7 is the coefficient of the 5th power.

ded to the second coefficient, the product of this sum by the root figure, added to the third coefficient, and so on.

EXAMPLE FOR THE BOARD.

7 times the 5th power, minus 2 times the 3d power, plus 5 times the second power of a certain number, is equal to 1405569.53125. What is the number?*

We commence with writing in order the coefficients, 7 for the 5th power, 0 for the 4th power,—2 for the 3d power, 5 for the 2d power, and 0 for the first power. Then, as the number has two integral periods, the first figure of the root will be tens. Finding by trial that 1 is the first root figure, we add 10×7 to 0; 10 ×70 to —2; &c., as in the following solution:

7	0	—2	5	0	1405569.53125(11.5
	70	700	6980	69850	698500
	70	698	6985	69850 c. d.	707069
	70	1400	20980	279650	426800
	140	2098	27965	(1) 349500 t. d.	280269.53125
	70	2100	41980	77300	280269.53125
	210	4198	(1) 69945	426800 c. d.	
	70	2800	7355	85019	
	280	(1) 6998	77300	(2) 511819 t. d.	
	70	357	7719	48720.0625	
	(1) 350	7355	85019	560539.0625	
	7	364	8090		
	357	7719	(2) 93109		
	7	371	4331.125		
	364	8090	97440.125		
	7	378			
	371	(2) 8468			
	7	194.25			
	378	8662.25			
	7				
	(2) 385				
	3.5				
	388.5				

The numbers are written in full, in order to add the respective coefficients.

* The equation stated algebraically, would be, $7x^5 - 2x^3 + 5x^2 = 1405569.53125$.

The first root figure in the following examples must be found by trial.

7. Five times the third power, plus three times the second power of a number, added to twice the number, make 9349968. What is the number?

8. A man being asked his age, replied: "If 3 times the square of my age be added to 5 times my age, the sum will be 2668." What was his age?

9. 7 times the 4th power+2 times the third power—13 times the second power of a certain number=2120912536. What is the number?

10. The 5th power of a certain number, diminished by 23 times the number, equals 14025514846. What is the number?

11. What is that number whose 5th power+5 times its 4th power—631 times its second power=426291377391?

12. What is that number 5 times whose 4th power+25 times its 3d power+125 times its 2d power=1364615.9375?

13. What is the number of miles from New York to Baltimore, if 13 times its 4th power+79 times its 3d power—93 times its 2d power,=17848292769?

14. The 6th power of a certain number exceeds its 2d power by 65944160560800. What is the number?

15. Extract the cube root of 3; the 5th root of 4; the 7th root of 5.

16. What is the value of x, if $5x^5+3x^4+17x=1603\text{-}5686129037$?

When two equal quantities are connected by the sign of equality, as in the above example, the whole is called an EQUATION.

A QUADRATIC EQUATION, is one in which the highest power of the unknown quantity is the square.

17. Find the value of x in the quadratic equation $3x^2+7x=780$.

18. Find the root of the quadratic equation $2x^2+5x=37125$.

19. What is the value of x, if $9x^2+13x=58006$?

20. What is the value of x in the quadratic equation, $4x^2+x=159.3275$?

CHAPTER XVII.

ARITHMETICAL PROGRESSION—OR, EQUIDIFFERENT SERIES.

A series of numbers, in which the successive terms increase or diminish uniformly by the same number, is called an ARITHMETICAL PROGRESSION,—PROGRESSION BY DIFFERENCE, or EQUIDIFFERENT SERIES. The difference between the successive terms, is called the *common difference*. The first and last terms of the series are called the *extremes*; the other terms the *means*.

Thus, in the ascending series,—

2, 4, 6, 8, 10, 12, 14, 16,

the extremes are 2 and 16, and the common difference is 2.

In the descending series,—

20, 16, 12, 8, 4, 0,

the extremes are 20 and 0, and the common difference is 4.

Any three of the five following things being given, the other two may be found:

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PROBLEM I.

One of the extremes, the common difference, and the number of terms being given, to find the other extreme and the sum of all the terms.

What is the tenth term of an ascending series, the first term being 1 and the common difference 4?

The second term will evidently be $1+4$; the 3d, $1+2\times4$; the 4th, $1+3\times4$, and so on to the 10th, which is $1+9\times4$ or 37.

What is the sum of the first ten terms of the above series?

To obtain a rule for finding the sum, we will invert the whole series, and write it under itself. In this manner we shall evidently obtain twice the sum of the series. And we may moreover observe, that *the sum of the extremes is equal to the sum of*

any two terms equally distant from the extremes, or to twice the middle term, when the number of terms is odd.

1,	5,	9,	13,	17,	21,	25,	29,	33,	37.
37,	33,	29,	25,	21,	17,	13,	9,	5,	1.
38,	38,	38,	38,	38,	38,	38,	38,	38,	38.

We thus have 10 times 38, or *the number of terms multiplied by the sum of the extremes*, which is equal to twice the sum desired. Hence we derive the following

RULE.

To obtain the last term, multiply the common difference by the number of terms less one, and if the series is ascending, add the product to the first term,—if decreasing, subtract it from the first term.

To obtain the sum of the terms, multiply the number of terms by the sum of the extremes, and divide the product by two.*

1. The first term of an equidifferent series is 5, the common difference 2, and the number of terms 11. Find the last term, and the sum of all the terms.

2. What is the sum of 45 terms of the natural series, 1, 2, 3, &c.?

3. Find the 23d term, and the sum of 23 terms of the series 1, 4, 7, 10, &c.

4. How many times does a clock strike in 12 hours?

5. The first term of a descending series is 30, the common difference $\frac{1}{2}$, and the number of terms 33. Required the last term, and the sum of the series.

6. If I travel 100 miles to-day, and 5 miles less on each succeeding day, how far shall I have gone at the end of the 17th day, and what will be the length of the last day's journey?

PROBLEM II.

The extremes and number of terms being given, to find the common difference.

The first term of an equidifferent series is 27, the last term 3, and the number of terms 9. What is the common difference?

The last term is found by Problem I. by subtracting 8 times

* If the given term is the *last term*, invert the series, and work by the same rule.

the common difference from the first term. Then the difference of the extremes 24, must be 8 times the common difference, which is therefore 3.

RULE.

Divide the difference of the extremes by the number of terms less one, and the quotient will be the common difference. This difference repeatedly added to the less, or subtracted from the greater term, will give the intermediate terms.

The sum of the terms is found by the preceding problem.

7. One hundred and fifty eggs were laid in a row. The first was 2 yards, and the last 100 yards from a basket. How far were the eggs apart, and how far must a person travel to pick them up singly and lay them in the basket?

8. A man had 15 children, whose ages were in Arithmetical Progression: the youngest was 3 years old, and the oldest 24. Required the common difference of their ages.

9. The extremes of an equidifferent series are 1 and 11, and the number of terms 25. What are the mean terms?

10. Insert 12 arithmetical means between 20 and 59. [As there are 12 means, the number of terms must be 14.]

11. Insert 13 arithmetical means between 3 and $4\frac{1}{2}$.

12. A man commenced walking for exercise, daily increasing the distance by equal additions. The first day he walked 4 miles, and on the 15th day he walked $7\frac{1}{2}$ miles. What was the daily increase?

PROBLEM III.

The extremes and common difference being given, to find the number of terms?

The extremes are 45 and 10, and the common difference is 5. What is the number of terms.

By Problem I. the difference of the extremes is equal to the common difference multiplied by the number of terms less one. Then the difference of the extremes 35, divided by the common difference 5, gives 7, which must be equal to the number of terms less 1. Therefore the number of terms is 8.

RULE.

Divide the difference of the extremes by the common difference, and add 1 to the quotient.

13. What is the sum of the series 2, 4, 6, 8, &c., to 1000?

14. What is the sum of the descending series 100, 99½, 99, 98½, &c., the last term being 0?

15. The first term of a series is 11, the last term 2, and the common difference $\frac{1}{7}$. What is the number of terms?

16. The first term 2½, the common difference $\frac{1}{8}$, and the last term 90, being given, required the number of terms.

PROBLEM IV.

The extremes and sum of the terms being given, to find the number of terms.

RULE.

Divide twice the sum of the terms by the sum of the extremes.

17. The sum of the terms is 22½, the first term 4, and the last term 11. Required the number of terms and the common difference.

PROBLEM V.

One of the extremes, the number of terms, and the sum of the terms being given, to find the other extreme.

RULE.

Divide twice the sum of the terms by the number of the terms, and subtract the given extreme from the quotient.

18. The sum of the terms in an equidifferent series is 27, one of the extremes is 4, and the number of terms 8. Required the other extreme and the common difference.

PROBLEM VI.

The number of terms, common difference, and sum of the terms being given, to find the extremes.

RULE.

Divide the sum of the terms by the number of terms. Then multiply the common difference by the number of terms less one, and subtract half the product from the first quotient for one extreme. For the other extreme add half the product to the quotient.

Problems IV., V., and VI., are all readily deduced from Problem I.

CHAPTER XVIII.

GEOMETRICAL PROGRESSION, OR CONTINUAL PROPORTIONALS.

A SERIES of numbers whose successive terms increase or diminish uniformly in the same ratio, is called a PROGRESSION BY QUOTIENT, or GEOMETRICAL PROGRESSION. The numbers may also be regarded as a series of CONTINUAL PROPORTIONALS. The *ratio* has already been defined in the Chapter on Proportion. As in Arithmetical Progression, the first and last terms are called the *extremes*, the other terms the *means*.

Thus in the ascending progression,—

$$1, 3, 9, 27, 81, 243,$$

the extremes are 1 and 243, and the ratio is 3.

In the descending progression,—

$$24, 12, 6, 3, \tfrac{3}{2}, \tfrac{3}{4}, \tfrac{3}{8}, \tfrac{3}{16},$$

the extremes are 24 and $\frac{3}{16}$, and the ratio is $\frac{1}{2}$.

From the nature of the series, it is evident that any four successive terms of a Geometrical Progression, constitute a proportion; as $1 : 3 :: 9 : 27$, $3 : 9 :: 27 : 81$, &c., in the first of the above series; $12 : 6 :: 3 : \frac{3}{2}$, $6 : 3 :: \frac{3}{2} : \frac{3}{4}$, &c. in the second series.

PROBLEM I.

One of the extremes, the ratio, and the number of terms being given, to find the other extreme.

The first term of an increasing geometrical series is 2, and the ratio 3. What is the sixth term?

The second term will be 2×3; the third, $2 \times 3 \times 3$ or 2×3^2; the fourth, $2 \times 3^2 \times 3$, or 2×3^3, and so on, to the sixth, which is 2×3^5. If the sixth term had been given, and the first required, we should evidently have been obliged to divide by 3^5.

RULE.

Raise the ratio to a power whose index is equal to the number of terms less one. Then for the last term multiply, and for the first term divide, the given extreme by this power of the ratio.

We have already seen in Involution, that by adding the exponents of two powers of the same number, we shall obtain the exponent of their product. Thus $3^7 \times 3^4 = 3^{11}$; $5^2 \times 5^3 = 5^5$, &c. This principle will greatly assist us in finding the powers of the ratio.

What is the 17th power of 2?

$\overset{1}{2}, \overset{2}{4}, \overset{3}{8}, \overset{4}{16}.$ $2^4 \times 2^4 = 256 = 2^8$. $2^8 \times 2^8 = 256 \times 256 = 65536 = 2^{16}$ $2^{16} \times 2 = 131072 = 2^{17}$.

In this instance we form the powers as high as the 4th power, then multiply the 4th power by itself for the 8th power,—the 8th power by itself for the 16th power,—and the 16th power by the 1st power for the 17th power. We have, therefore, after obtaining the 4th power, only *three* multiplications to make, instead of *thirteen*, which would otherwise have been necessary.

1. The first term of a geometrical series is 3, the ratio 2, and the number of terms 9. What is the last term?

3. What is the 11th term of the series 4096, 1024, 256, &c.?

3. What is the 7th term of a series, whose first term is 20, and ratio 1.06?

4. What is the amount of $20 for 7 years, at 6 per cent. compound interest?

5. What is the amount of $300 for 9 years, at 6 per cent. compound interest?

6. If the first pane of glass in a window cost 1 mill, the second 2 mills, the third 4 mills, the 4th 8 mills, &c., what would be the price of the twelfth pane?

7. If $100 were at simple interest for three years at 6 per cent., and the amount then placed at compound interest at 5 per cent., what would be the whole amount at the expiration of 12 years?

8. The first term is 8, the ratio $\frac{1}{8}$, and the number of terms 8. What is the last term?

9. The twelfth term is 59049, and the ratio $\frac{1}{3}$. What is the first term?

10. What principal will amount to $4489.643, in 12 years, at 5 per cent. compound interest? +

11. If a farmer plants a grain of wheat, and each year plants the product of the preceding harvest, how much will

he harvest in the 15th year, the annual increase being 12 fold?

12. A farmer inquiring the price of a drove of 30 oxen, was told that he might have the whole drove for the price of the 20th ox, valuing the first at one cent, the second at 2 cents, the third at 4 cents, and so on, doubling the price of each ox for the price of the next. What would be the price per head at that rate?

13. What is the amount of $275 for 9 years at 4 per cent. compound interest?

14. What sum would amount to $300 in 10 years at 5 per cent. compound interest?

15. What sum of money would amount to $1000 in 12 years at 6 per cent. compound interest?

16. What sum would amount to $2500 in 3 years at 8 per cent. compound interest?

17. What would be the amount of $2300 in 13 years at 7 per cent. compound interest?

18. The first term of a geometrical series is 4194304, and the ratio $\frac{1}{4}$. What is the fourteenth term?

PROBLEM II.

The extremes and ratio being given, to find the sum of the terms.

The first term of a series is 162, the last term is 2, and the ratio $\frac{1}{3}$. What is the sum of the series?

The series is 162, 54, 18, 6, 2, the sum of which is 242.

54, 18, 6, 2, $\frac{2}{3}$, is another series obtained by multiplying each term of the first series by the ratio. Subtracting the second series from the first, we have $161\frac{1}{3}$, which is the difference between the first term and the last term multiplied by the ratio. It is also $\frac{2}{3}$ of the sum of the first series, since it is obtained by subtracting $\frac{1}{3}$ of the series from the whole series. Then if we divide by $\frac{2}{3}$, or the difference between the ratio and 1, we shall obtain the desired sum.

RULE.

Multiply the last term by the ratio, and divide the difference between the product and the first term by the difference between the ratio and 1.

19. The first term 9, the ratio $\frac{1}{3}$, and the last term $\frac{1}{243}$, are given. What is the sum of the series?

20. What is the sum of 12 terms of the progression 2, 8, 32, &c.? (The last term is found by Problem I.)

21. What is the sum of the series 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, &c. to infinity? (The last term in any infinite decreasing series is 0.)

22. Required the sum of the infinite series 7, $\frac{7}{3}$, $\frac{7}{9}$, &c.?

23. If I lay up $100 every year, to what will the whole amount in 10 years, at 6 per cent. compound interest? (The 1st term is 100, the ratio 1.06, and the number of terms 10.)

24. What would be the amount of an annual saving of $300 for thirty years, at 6 per cent. compound interest?

25. Sysla, the reputed inventor of the game of chess, is said to have asked as a reward, one grain of wheat for the first square on the chess-board, two for the second, and so on in geometrical progression. What would have been the amount of his reward, there being 64 squares on the board, and 9200 grains of wheat in a pint? What would be the height of a cubical bin that would contain it, supposing the base to be 10 miles square?

26. A blacksmith agreed to shoe a horse for the amount of 32 nails, at 1 mill for the first nail, 2 mills for the second, 4 for the third, and so on. What was his charge?

27. What is the sum of the infinite series 6, $1\frac{1}{2}$, $\frac{3}{8}$, $\frac{3}{32}$, &c.?

28. What is the sum of 20 terms of the series 10, 5, $2\frac{1}{2}$, $1\frac{1}{4}$, &c.?

29. If a man commences at 21 years of age, and annually puts $500 at compound interest, how much will he be worth when he is 50 years old?

PROBLEM III.

The extremes and ratio being given, to find the number of terms.

RULE.

Divide the last term by the first, and add one to the index of that power of the ratio, which is equivalent to the quotient.

30. A man made several payments in Geometrical Progression, each being twice as large as the preceding. The first payment was $4, and the last $512. What was the number of payments, and what was the whole amount of the debt?

PROBLEM IV.

The extremes and number of terms being given, to find the ratio.

RULE.

Divide the last term by the first, and extract the root of the quotient which is indicated by the number of terms less one.

This and the preceding rule, are readily deduced from Problem I.

31. The first term in a Geometrical series is 1, and the eleventh term is 1024. What is the ratio?

32. The first term is 256, and the fifth term 81. What is the ratio?

33. The first term is 2, the last term $\frac{64}{3125}$, and the number of terms 6. What is the ratio?

34. An estate of $200 amounted to $264.86, in seven years at compound interest. What was the rate per cent?

35. At what rate per cent. will $1000 amount to $1689.48, in nine years at compound interest?

36. Insert 2 mean proportionals between 1 and 6. (As there are to be 2 means, the number of terms is 4, and the extremes 1 and 6.)

37. Insert 4 mean proportionals between 3 and 96.

38. Insert 5 mean proportionals between 4 and 2916.

39. Insert 3 mean proportionals between 5 and 6480.

40. Insert 4 mean proportionals between 1 and 4.

The remaining Problems in Geometrical Progression, are of little interest.

By observing the formulas on the following page, we shall perceive a striking analogy between Arithmetical and Geometrical Progression. The continual product of all the terms in a Geometrical series, is denoted by p.

Geometrical Progression.	*Arithmetical Progression.*
$l = a \times r^{n-1}$	$l = a + (n - 1)\, d$
$r = \sqrt[n-1]{\frac{l}{a}}$	$d = \frac{l - a}{n - 1}$
$p = \sqrt{(a\, l)^n}$	$s = \frac{(a + l)\, n}{2}$

Thus in comparing the first table with the second, we see that *multiplication* corresponds to *addition;*

division " " *subtraction;*
involution " " *multiplication;*
evolution " " *division.*

If, therefore, we had a series of numbers bearing the same ratio to the natural series, as an Arithmetical to a Geometrical Progression, the labor of multiplication would be reduced to that of simple addition, and involution to simple multiplication. Such a series constitutes a TABLE OF LOGARITHMS.

CHAPTER XIX.

HARMONICAL PROGRESSION.*

WHEN three numbers are such that the first is to the third, as the difference of the first and second is to the difference of the second and third, they are said to be in HARMONICAL PROPORTION, and a series of numbers in continued harmonical proportion, constitutes a HARMONICAL PROGRESSION.

The *reciprocal* of a number, is the quotient of 1 by the number. Thus $\frac{1}{2}$ is the reciprocal of 2; 4 is the reciprocal of $\frac{1}{4}$; $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, &c. *The reciprocals of any equidifferent series form a harmonical proportion.*

PROBLEM I.

Two numbers being given to find a third in harmonical proportion.

* So called, because if a musical string be divided in harmonical proportion, the different parts will vibrate in unison.

RULE.

Consider the reciprocals of the numbers as two terms of an equidifferent series. The third term will be the reciprocal of the number sought.

Find a third harmonical proportional to 120 and 40.

The reciprocals are $\frac{1}{120}$, and $\frac{1}{40}$ or $\frac{3}{120}$. The third term of the equidifferent series is $\frac{5}{120}$, and its reciprocal 24 is the harmonical proportional sought.

1. The first two terms of a harmonical progression are 60 and 30. Required the ten succeeding terms.

2. The first two terms of a harmonical proportion are 348075 and 69615. Find the six succeeding terms.

PROBLEM II.

To insert any number of harmonical means between two numbers.

RULE.

Find as many arithmetical means between the reciprocals of the given numbers. These means will be the reciprocals of the harmonical means.

Insert 4 harmonical means between 20 and 120.

The reciprocals are $\frac{1}{24}$ and $\frac{1}{120}$, or $\frac{6}{120}$ and $\frac{1}{120}$. The four arithmetical means are $\frac{5}{120}$, $\frac{4}{120}$, $\frac{3}{120}$ and $\frac{2}{120}$, whose reciprocals are 24, 30, 40 and 60,—the desired harmonical means.

3. Insert 7 harmonical means between 630 and 5040.

4. Insert 8 harmonical means between 10 and 60.

5. Insert 2 harmonical means between $\frac{1}{2}$ and $\frac{1}{3}$.

6. Insert 4 harmonical means between $\frac{1}{6}$ and $\frac{1}{10}$.

CHAPTER XX.

ANNUITIES.

Any sum of money to be paid regularly, at stated periods, is called an Annuity. The payment may be stipulated for

a given number of years, in which case it is called an *annuity certain*, or it may be dependent upon some particular circumstance, as the life of one or more individuals. The latter is called a *contingent annuity*. A *perpetual annuity*, is one which can only be terminated by the grantor, on the payment of a sum whose interest will be equivalent to the annuity. Of this character is the consolidated debt of England.

An *annuity in possession*, is one on which there is a present claim; an *annuity in reversion*, or deferred annuity, is one that does not commence until the lapse of a stated time, or the occurrence of some uncertain event, as the death of an individual.

The *present worth* of an annuity, is the sum which, at compound interest for the time of its duration, would amount to the sum of all the payments, each being placed at compound interest as it became due.

PROBLEM I.

To find the amount due on an annuity which has remained unpaid a given time.

As the payments are all at compound interest, this case evidently falls under Problem II., in Geometrical Progression.

RULE.

Find the sum of a Geometrical Progression, in which the first term is the annuity, the ratio is the amount of $1.00 *for the time that should elapse from one payment to another, and the number of terms is the number of payments due.*

1. If a person saves $250 per annum, and invests it at 7 per cent. compound interest, how much will he be worth at the end of 25 years?

2. What is the amount of an annuity of $500 payable semi-annually, forborn for 7 years, at 6 per cent. per annum?

3. What is the amount of a quarterly annuity of $400, in arrears for 5 years, at 5 per cent.?

4. What is the amount of an annual salary of $1000 for 10 years, at 6 per cent.?

5. What is the amount of a quarterly rent of $200 for 3 years, at 6 per cent.?

6. What is the amount of a pension of $400 a year, payable semi-annually, for 3 years and 6 months, at 7 per cent. per annum?

7. What is the amount of a rent of $1500, ten years forborn, at 5 per cent. per year?

8. An estate that yields an annual income of $2000, is offered for sale for the amount of 10 years income at 6 per cent. compound interest. What is the price of the estate?

9. What is the amount of a salary of $1300 for 16 years, at 5 per cent. compound interest?

PROBLEM II.

To find the present worth of an annuity certain.

What is the present worth of a semi-annual rent of $500, for 6 years at 6 per cent?

By the preceding Problem, we find that the rent at the expiration of the 6 years, will have amounted to $7096.015. The question is, therefore, to find the present worth of $7096.015 due in 6 years, at a compound interest of 3 per cent. semi-annually, which is done by Problem I., in Geometrical Progression.

RULE.

Divide the amount of the annuity, (found by Problem I.) by the amount of $1.00 for the same time.

10. What is the present worth of an annual salary of $800 to continue 8 years, at 5 per cent. compound interest?

11. What is the present worth of an annual income of $500, to continue 11 years, at 6 per cent. compound interest?

12. What is the present worth of a quarterly rent of $200, to continue 5 years, at 7 per cent. compound interest?

13. What is the present worth of a semi-annual pension of $175, to continue 9 years, at 4½ per cent. compound interest?

14. What is the present worth of an annuity of $3000 for 7 years, at 3 per cent. compound interest?

15. What sum invested at 6 per cent. compound interest, will yield me an income of $1600 per annum, for 25 years?

16. What sum at 5 per cent. will yield an annual income of $1200 for 15 years?

17. A gentleman wishes to present his estate to his children, reserving enough to yield $700 per annum for 15 years. How much must he reserve, allowing 5 per cent. compound interest?

PROBLEM III.

To find the present worth of a perpetual annuity.

The present worth of a perpetual annuity, is a sum which would yield an interest equivalent to the annuity. As the interest is found by multiplying the principal by the rate, the principal may be found by dividing the interest by the rate.

RULE.

Divide the annuity by the rate per cent.

18. For how much should an estate that rents for $175 per year, be sold, to allow the purchaser 6 per cent. interest on his investment?

19. What is the par value of an annual income of £500 in the 4 per cent. consols.* ?

20. What sum of money must be laid out in the 3 per cent. consols., at 68 per cent. of the par value, to yield an income of £1000.

21. What sum of money at 4½ per cent. will yield an annual interest of $620?

22. What sum, invested in an estate that rents for $400 per annum will yield an interest of 8 per cent.?

23. A farm rents for $750 per annum. For what price should it be sold, when money is worth 6 per cent. a year?

24. What sum will build a wall worth $1000, and renew it every 15 years, at 5 per cent. compound interest?

* Consols., is an abbreviation for the consolidated annuities of the British National Debt.

25. A railroad has been constructed through a farm, in consequence of which, the owner of the estate is obliged to expend $400 in fencing, that must be renewed at the expiration of every 12 years. What sum should he now receive, to compensate him for the required expenditure, money being worth 6 per cent. compound interest?

PROBLEM IV.

The present worth of a certain annuity being given, to find the annuity.

RULE.

Find the first term of a Geometrical Progression, in which the sum is the amount of the annuity for the whole time, the ratio is the amount of $1.00 *for the time that elapses between two successive payments, and the number of terms is the number of payments.*

26. The present debt of Pennsylvania (in 1844) amounts to about $40000000, which bears an interest of 5 per cent. per annum, payable semi-annually. What semi-annual appropriation will extinguish the debt in 40 years?

27. What sum of money must a man lay up annually, to amount to $10000 in 20 years, the investments being all made at 6 per cent. compound interest?

28. A builder takes a lease of a lot of ground for 25 years, and erects buildings on it which cost him $20000. Allowing money to be worth 6 per cent. compound interest, what *clear** annual rent must he receive from the buildings to reimburse his expenditure, at the termination of the lease,—the rent commencing one year after the lease is given?

29. The executors of an estate wish to dispose of an unexpired lease that has 8 years to run, for a premium of $1500. What amount must be added to the annual rent, for that purpose?

* The *clear* annual rent, is the amount received after deducting ground-rent, taxes, and other expenses.

PROBLEM V.

To find the present worth of an annuity in reversion.

RULE.

Find the present worth of the annuity until the commencement of the reversion, and also the present worth until its termination. The difference of these two values will give the present worth of the reversion.

30. What is the present worth of an annuity of $1100, to commence in 3 years and continue for 8 years, interest at 6 per cent?

31. What is the present worth of a perpetual annuity of $300, to commence in 2 years, at 5 per cent. interest?

32. A father leaves an annual rent of $400 to his eldest child for 5 years, and the reversion of it for the 8 succeeding years to his youngest child. What is the present worth of each legacy, at 7 per cent.?

33. What sum must be paid, allowing 6 per cent. compound interest, to extend a lease 7 years,—the clear annual rent being $500, and the lease having 4 years to run?

34. What is the value of $3000 rail-road stock, that will yield no income for 4 years, but on which, after that time, there will be an annual dividend of 9 per cent. for 21 years?

35. What is the present worth of a reversion of $700 per annum, to commence in 20 years, and continue 40 years thereafter, allowing 6 per cent. compound interest?

The operations in annuities are generally so tedious, that tables similar to those given at the end of the book, are employed in their calculation.

The subject of Life Annuities, involves the calculation of chances, as applied to the duration of life. Tables of the probable duration of life at each year, are prepared, and the number against each age is employed as a multiplier or divisor, for determining the present worth, or the annuity itself.

CHAPTER XXI.

EXCHANGE.

THE term EXCHANGE in commerce, is generally employed to designate that species of mercantile transactions, by which the debts of individuals residing at a distance from their creditors, are cancelled without the transmission of money.

A BILL OF EXCHANGE is an order addressed to some person at a distance, directing him to pay a certain sum to the person in whose favor the bill is drawn, or his order. The person who draws the bill, is called the *drawer;* the person in whose favor it is drawn, the *remitter* or *payee;* the person on whom it is drawn, the *drawee.* The drawer is also called the *acceptor*, when he has accepted, or engaged to pay the bills.

Though Bills of Exchange are originally drawn by creditors on their debtors, they are very rarely transmitted directly, but pass from hand to hand like any other circulating medium, and are bought and sold in the market. When the remitter disposes of a bill, he writes his name on the back, and is termed the *indorser.* If he indorses in favor of any particular individual, he gives a *special indorsement*, and the *indorsee* must also indorse the bill if he negotiates it. But if the indorsement is blank, the bill may be passed at pleasure from hand to hand. Every indorser, as well as the acceptor, is held responsible for the payment of the bill, and may be sued for its recovery.

INLAND, or DOMESTIC EXCHANGE, includes the commercial transactions within the limits of one country. FOREIGN EXCHANGE relates to the transactions of one country with another.

The TRUE PAR OF EXCHANGE is the value of the currency of one country estimated in the currency of another, by comparing the quantity of gold and silver in their respective coins. The exchange with England apparently furnishes an exception to this rule, the nominal par being $\$4.44\frac{4}{9}$ per £, while the actual value of the pound sterling, which is the

real par, is about $4.87. Hence exchange on England is generally said to be from 8 to 10 per cent. above par.

The COURSE OF EXCHANGE, or the fluctuation above or below par, depends generally on the amounts due between different countries. Thus when the debts and credits between two countries are equal, the real exchange is at par. But if New York owes London more than London owes New York, there will be a greater demand for bills on London, and this demand will produce a rise in the price, or cause the bills to be at a *premium* The premium, however, can never exceed the cost of transporting specie; for if it did, all debts would be paid in money or merchandise, instead of bills of exchange. The *nominal* premium, however, may exceed the cost of remitting coin, when the nominal par is above the real par.

The operation of Bills of Exchange, may be explained by a single example.

If A. of Boston, owes B. of Paris, and C. of Paris owes D. of Boston, A. purchases in the market a *bill* upon Paris; that is, he buys of D. an order on his creditor C., to pay A. or his order, the amount desired. A. indorses the bill, and sends it to B., who receives payment from C. Thus the two debts are cancelled by a single remittance; the inconvenience of exporting and re-importing coin is removed, and all danger of loss is obviated, by sending three bills, (called the First, Second, and Third of Exchange,) either of which being paid the others are void.

An ACCEPTANCE is an engagement to pay the amount of the bill, and may be either *absolute* or *qualified.* An absolute acceptance binds the drawer when the bill becomes due, and in making it, the drawee usually writes "Accepted," and subscribes his name at the bottom, or across the body of the bill. A qualified acceptance implies some condition, as the sale of merchandise, &c., and does not bind the acceptor until the condition is complied with. If a bill is made payable at a certain time after sight, the acceptance should be dated.

A bill should be presented for payment during the regular hours of business, on the day it becomes due.

When acceptance or payment has been refused, the holder should give immediate notice to all the parties whom he in-

tends to hold responsible for the payment of the bill. This notice is usually accompanied with a PROTEST, which is an instrument prepared by a public notary, stating that acceptance or payment has been demanded and refused, and that the holder of the bill intends to recover any damages which he may sustain in consequence.

In some places on the continent of Europe, banks of deposite are established, and exchanges are frequently made by transferring the amounts credited on the books of the bank from one person to another. The deposites on which these credits are based, are called *banco*, and they usually bear a premium above the ordinary currency of the country. This premium is called the *agio*.

The comparative market value of gold and silver is constantly varying, and the mint value is differently estimated by different governments. Thus, in England the relative worth of the two metals is as 1 to 14.29, in France, as 1 to 15.52, and in the United States, as 1 to 15.99. In England, silver is so much overvalued, that it would banish the gold coins from circulation, were there not a statute providing that *gold only shall be legal tender in all payments of more than* 40 *shillings*. The relative value of the precious metals should always be considered, in estimating the true par of exchange with any country.

DOMESTIC EXCHANGE.

Inland Exchange is usually effected by checks or DRAFTS, similar in form to the following:

$1275.25 Philadelphia, June 3, 1844.

Sixty days from date, pay to James N. Lewis, or order, Twelve Hundred and Seventy-Five Dollars and Twenty-Five Cents, and charge the same to my account.

William Morris.

To Markham & Jones,
Merchants, Cincinnati.

The premium or discount on drafts, may be owing either to a difference in the value of the circulating medium, or to fluctuations in the demand.

The English denominations of shillings and pence, are

still retained in this country to some extent. At the formation of the Constitution, the continental currency had suffered a greater depreciation in some of the colonies than in others. Thus, while a pound in New England was worth \$3.33$\frac{1}{3}$, in Pennsylvania it was but \$2.66$\frac{2}{3}$, and in New York but \$2.50. The value in Federal Money, of the old currencies of the different States, is as follows:

A shilling of New England, Virginia, Kentucky, or Tennessee, is 16$\frac{2}{3}$ cents.

A shilling of New York or North Carolina, is 12$\frac{1}{2}$ cents.

A shilling of New Jersey, Pennsylvania, Delaware, or Maryland, is 13$\frac{1}{3}$ cents.

A shilling of South Carolina, or Georgia, is 21$\frac{3}{7}$ cents.

CHAPTER XXII.

DIVISIBILITY OF NUMBERS.

Every number that cannot be divided by any other number, (except 1,) without a remainder, is called a PRIME NUMBER.

Two or more numbers that have no common divisor, are said to be *prime to each other*. Every prime number is prime to all other numbers except its own multiples.

There are no known means of determining at once whether a proposed number is a prime, but the following properties and rules will enable us to determine all the divisors of any number.

1. 2 is a factor of all numbers terminated by 0, 2, 4, 6, or 8. For, as 2 will divide 10, it will also divide any number of tens, or any number of tens *plus* 2, 4, 6, or 8. Numbers divisible by 2 are called EVEN,—all others, ODD numbers.

2. 5 is a factor of all numbers terminated by 0 or 5. For, as 5 will divide 10, it will also divide any number of tens, or any number of tens *plus* 5.

3. 3, or 9, is a factor of all numbers in which the sum of the figures is exactly divisible by 3, or 9. For, if from

any power of 10, as 10, 100, 1000, &c., we subtract 1, the remainder consists entirely of 9's, and is, therefore, divisible by both 3 and 9. Hence, any power of 10 is divisible by 3 and 9 with 1 remainder, therefore, any number of tens, hundreds, thousands, &c., diminished by as many units, will be divisible by 3 and by 9. Let us, then, examine the number 34794. 3 ten thousands — 3; 4 thousands — 4; 7 hundreds — 7; 9 tens — 9; and 4 units — 4; each divided by 3 or 9, give no remainder. Therefore, 34794 — 3 — 4 — 7 — 9 — 4, is divisible by 3 and by 9, and if the sum of the numbers subtracted, or in other words, the sum of the digits, is similarly divisible, the number itself will be so.

4. 11 is a factor of all numbers in which the sum of the odd digits, (the 1st, 3d, 5th, &c.,) and the sum of the even digits, (the 2d, 4th, 6th, &c.,) are equal, or their difference is some multiple of 11. For any number of tens, thousands, hundred thousands, &c., (which represent the even digits,) increased by as many units, will be divisible by 11. Any number of hundreds, ten thousands, millions, &c., (which represent the odd digits,) diminished by as many units, will also be divisible by 11. Take, then, the number 635173. 6 hundred thousands + 6; 3 ten thousands — 3; 5 thousands + 5; 1 hundred — 1; 70 + 7; and 3 — 3; each divided by 11 give no remainder. Therefore, 635173 — 18 + 7 or 635173 — 11, is divisible by 11, and 635173 itself must be so.

5. 4 is a factor of all numbers, in which the two terminating figures are divisible by 4. For, as 4 will divide 100, it will also divide any number of hundreds, or any number of hundreds *plus* any number of units divisible by 4.

6. 25 is a factor of all numbers terminated by 25, 50, 75, or two zeroes. For, as 25 will divide 100, it will also divide any number of hundreds, or any number of hundreds *plus* 25, 50, or 75.

7. Every number that is divisible by two or more numbers prime to each other, is divisible by their product. Take for example, 105 which is divisible by both 3 and 5. This number may be resolved into the factors 5 × 21; 5 × 21 must, therefore, be divisible by 3. But as 3 will not divide

5, it must divide the other factor 21, and the number may be resolved into the factors $5 \times 3 \times 7$ or 15×7. Hence we deduce the following additional properties.

8. Every even number that is divisible by 3 is also divisible by 6; and every even number that is divisible by 9 is also divisible by 18.

9. Every number divisible by 3 or 9, in which the two terminating figures are divisible by 4, is divisible by 12 or 36.

10. Every number divisible by 3 or 9, whose terminating digit is 0 or 5, is divisible by 15 or 45.

11. Every prime number greater than 2, is one greater or one less than some multiple of 4.

12. Every prime number greater than 3, is one greater or one less than some multiple of 6.

13. Every number that has no prime factor, equal to, or less than its square root, is itself a prime number. For the product of any two factors, each greater than the square root of a number, would evidently be greater than the number itself. *Therefore, if we attempt the division of any supposed prime, by all the primes less than its square root, and discover no factor, the number is itself a prime.*

TO FIND ALL THE DIVISORS OF A NUMBER.

What numbers will divide 5940 without a remainder?

2	5940
2	2970
3	1485
3	495
3	165
5	55
11	11
	1

We first resolve the number into all its prime factors, by commencing with 2 and dividing as often as possible, by each of the prime numbers in succession. We thus find that $41580 = 2^2 \times 3^3 \times 5 \times 11$, or $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 11$. It may, therefore, have as many composite divisors as we can form distinct products of these prime factors. In order to determine all the possible products, we arrange 1, with the powers of the factor that is employed the greatest number of times, in a horizontal line. We then multiply each of the numbers in the first line, by each of the powers of another factor,—each of the numbers of the preceding lines, by each of the powers of a third factor, &c., as in the following table.

1	3	9	27	$= 3^3$
2	6	18	54	$= 3^3 \times 2$
4	12	36	108	$= 3^3 \times 2^2$
5	15	45	135	$= 3^3 \times 5$
10	30	90	270	$= 3^3 \times 2 \times 5$
20	60	180	540	$= 3^3 \times 2^2 \times 5$
11	33	99	297	$= 3^3 \times 11$
22	66	198	594	$= 3^3 \times 2 \times 11$
44	132	396	1188	$= 3^3 \times 2^2 \times 11$
55	165	495	1485	$= 3^3 \times 5 \times 11$
110	330	990	2970	$= 3^3 \times 2 \times 5 \times 11$
220	660	1980	5940	$= 3^3 \times 2^2 \times 5 \times 11$

The numbers of the first line having been arranged as directed, we multiply them separately by 2 and 2^2.

All the numbers of these *three* lines, are multiplied by 5, which gives us three new lines of divisors.

All the numbers of these *six* lines, are multiplied by 11, which gives us six new lines of divisors. We thus obtain 48 numbers that will divide 5940 without a remainder, and an examination of the table will show that these are *all* the divisors, since the prime factors are combined in every possible way.

We are able to determine without actual trial, the number of exact divisors of any given number. By the foregoing table we perceive that 3^3 had 4, or $3 + 1$ divisors. $3^3 \times 2^2$ has 12, or $\overline{3 + 1} \times \overline{2 + 1}$. $3^3 \times 2^2 \times 5$ has 24 or $\overline{3 + 1} \times \overline{2 + 1} \times \overline{1 + 1}$. In like manner each new factor can be multiplied by all the preceding divisors, as many times as are equivalent to the exponent of its power, thus forming so many new divisors, to be added to the preceding. Hence, for finding the number of divisors of any given number, we have the following

RULE.

Add 1 to the exponent of each of the prime factors of the given number, and multiply together the exponents thus increased. The product thus obtained, is the number of divisors sought.

We have already seen that the greatest common divisor of two or more numbers, may be readily obtained by the aid of a table of prime factors. But by resolving fractions by inspection, into their prime factors, we may often reduce them to their lowest terms, without finding the greatest common divisor. For example, let it be required to reduce

$\frac{68}{85}$, $\frac{105}{132}$, $\frac{279}{341}$, and $\frac{385}{1045}$, to their lowest terms.

Resolving each fraction into its prime factors, we have

$$\frac{4 \times 17}{5 \times 17}, \frac{3 \times 5 \times 7}{3 \times 4 \times 11}, \frac{3 \times 3 \times 31}{11 \times 31}, \text{ and } \frac{5 \times 7 \times 11}{5 \times 11 \times 19}.$$

Cancelling the factors common to the numerators and denominators, we have $\frac{4}{5}$, $\frac{35}{44}$, $\frac{9}{11}$, $\frac{7}{19}$ for the lowest terms of each fraction.

1. Resolve 65340 into its prime factors.
2. Find all the divisors of 1200; of 1620.
3. How many integral divisors has 1844?
4. How many integral divisors has 1900?
5. Reduce $\frac{4915}{10813}$ to its lowest terms.
6. Is 479 a prime number?
7. Is 30907 a prime number?
8. Reduce $\frac{1154}{1731}$ to its lowest terms.
9. Reduce $\frac{542}{2710}$ to its lowest terms.
10. How many integral divisors has 13600?
11. How many integral divisors has 13475?
12. What are the integral divisors of 700?
13. What are the integral divisors of 1584?
14. What are the integral divisors of 2310?
15. What are the prime factors of 1770?
16. What are the prime factors of 13470?
17. How many integral divisors has 95875?
18. Reduce $\frac{706}{10943}$ to its lowest terms.
19. Reduce $\frac{8261}{12767}$ to its lowest terms.
20. Reduce $\frac{1335}{1958}$ to its lowest terms.
21. Is 21479 a prime number?

22. Is 52099 a prime number?

23. How many intégral divisors has 57660?

24. What are the prime factors of 168432?

CHAPTER XXIII.

NUMERICAL APPROXIMATIONS.

THE student will have already perceived, in circulating decimals, and the extraction of surd roots, that there are many arithmetical operations which never give an exact result. There are also others, which, by a tedious process, would furnish an exact answer, but in which we desire only an approximate value, and we would gladly know what part of our labor may be omitted without affecting the accuracy required. A few of the most important NUMERICAL APPROXIMATIONS will form the subject of the present chapter.

In ADDITION and SUBTRACTION of circulating decimals, it has been recommended to continue the repetends to five or six figures. If we wish to obtain the exact repetend, it will be necessary to *change all the given repetends into others, containing as many figures as the least common multiple of the number of places in each repetend.*

Add 17.5, $3.\dot{7}$, $419.0\dot{8}7\dot{5}$, $1.985\dot{6}\dot{3}$, and $32.\dot{1}27\dot{8}$.

The numbers of repetend figures are 0, 1, 3, 2, and 4; the least common multiple of which is 12. The common repetend must therefore consist of 12 figures, commencing at the lowest place of the given repetends, which is *ten-thousandths*. The numbers will be written as follows.

$$
\begin{array}{r}
17.500\dot{0}0000000000\dot{0} \\
3.777\dot{7}7777777777\dot{7} \\
419.087\dot{5}8758758758\dot{7} \\
1.985\dot{6}3636363636\dot{3} \\
32.127\dot{8}1278127812\dot{7} \\
\hline
474.478\dot{8}1451027985\dot{6}
\end{array}
$$

Adding the repetends, we find their sum is $\frac{2814510279854}{999999999999}$. Dividing by the rule for division by nines, we obtain a quotient $2\dot{8}1451027985\dot{6}$. The repetend is written down, and the 2 carried to the column of thousandths. Repetends, that thus commence and end at the same decimal places, are called *similar and conterminuous.*

Subtract $1.7\dot{4}\dot{2}$ from $2.\dot{9}3\dot{7}$.

$2.9\dot{3}7937\dot{9}$	The common repetends have 6 figures. From
$1.7\dot{4}2424\dot{2}$	the right-hand figure of the remainder we subtract
$1.1\dot{9}5513\dot{6}$	1, because the repetend of the minuend is less

than that of the subtrahend. The reason of this subtraction will become evident, if we change the repetends to fractions, and subtract $1.7\frac{424242}{999999}$ from $2.9\frac{379379}{999999}$

1. Add $17.6\dot{9}$, $18\dot{3}$, 25.75, $3.\dot{2}7\dot{6}$, $194.\dot{4}\dot{3}$, and 649.287.
2. Add $3.2\dot{1}\dot{9}$, $63.\dot{3}7\dot{4}$, $285.\dot{1}\dot{2}$, 38.4, $.0\dot{3}7\dot{1}$, and $43.6\dot{8}$.
3. Subtract $49.\dot{2}87\dot{1}$ from 64.
4. Subtract $215.\dot{9}93\dot{1}$ from $1842.\dot{2}43\dot{4}$.
5. Subtract 11.27 from $30.\dot{4}0\dot{9}$.
6. Subtract $2856.\dot{0}3\dot{6}$ from $3017.\dot{6}259\dot{1}$.
7. Subtract $43.76\dot{3}$ from $288.1\dot{9}5\dot{4}$.
8. Add $21.\dot{3}$, 28.72, $6.\dot{4}\dot{7}$, $19.3\dot{4}\dot{5}$, $201.1\dot{5}9\dot{3}$, and $419.\dot{6}6243\dot{4}$.
9. Add 7.83, $24.\dot{1}$, $79.1\dot{4}\dot{2}$, $252.\dot{4}16\dot{3}$, and $17.3\dot{0}8\dot{7}$.
10. Subtract $4.1\dot{9}5\dot{6}$ from $21.28\dot{4}3911\dot{3}$.

In Multiplication, if only a certain degree of accuracy is desired, the product may be obtained by writing the units' figure of the multiplier under that figure of the multiplicand, whose place we would reserve in the product, and inverting the order of the remaining figures. In multiplying, we commence, for each partial product, with the figure of the multiplicand immediately above the multiplying figure, carrying the tens, which would arise from the multiplication of the two rejected figures at the right.

Required the product of 287.613952 by 15.98421, correct to the fourth decimal place.

287.613952	287.613952	
12489.51	15.98421	
2876.1395	28	7613952
1438.0698	575	227904
258.8525	11504	55808
23.0091	230091	1616
1.1505	2588525	568
575	14380697	60
29	28761395	2
4597.2818	4597.2818	0769792

The units' figure of the multiplier being placed under the 4th decimal of the multiplicand, and the whole multiplier reversed, the product of each figure by the one above it will be ten-thousandths. Therefore the right-hand figure of each partial product, will fall in the column of ten-thousandths. In the second product, multiplying 52 by 5 we obtain 260, which being nearer 300 than 200, we carry 3 to the product of 9 by 5.

The multiplication has also been performed in the usual way, the vertical line showing the figures that are rejected.

If the multiplicand does not contain enough decimal figures to correspond with the inverted multiplier, the deficiency should be supplied by annexing zeroes. The same contraction may be applied to integers, if we wish only to obtain the thousands, millions, &c., of the product.

11. Required the product of 2869.174381 by 154.49216, true to three places of decimals.

12. Find the product of 176.2428 by 119.43, true to the second decimal place.

13. What is the integral part of the product of 49821.476 by 25.341?

14. What is the integral part of the product of 51763. 84926 by 2.4957?

15. Multiply 778148.3219 by 954.638, and reserve two decimal places.

16. Multiply 11817.93642 by 2581.36, and reserve two decimal places.

17. Multiply 4435.81977 by 6.9043, and reserve one decimal place.

18. What is the product of 7716.4295 by 19.87436, within .001?

19. Find the product of 63917.48219 by 587.618, within one *ten-thousandth.*

20. Find the product, true to the 4th decimal place, of 21.87964 by 2.38917.

In Division, a similar contraction may be made when the the divisor is large, which is also applicable in the extraction of roots.

The first quotient figure is of the same numerical value as

the figure of the dividend which stands immediately over the units of the divisor, at the first step of the division.

After the first remainder has been obtained, instead of bringing down the remaining figures of the dividend, we may cut off the right-hand figure of the divisor at each step, as in the following example.

342.15)28417.95255(83.057	342.15)28417.9|5255(83.057
27372 0	27372 0|
10459	10459|5
10264	10264|5
195	195|025
171	171|075
24	23|9505
24	23|9505

In the complete division the contraction is indicated by the vertical line. In each multiplication, the tens arising from the product of the quotient figure by the suppressed figure of the divisor, must always be carried as in contracted multiplication.

The right-hand figure of the quotient thus obtained, cannot always be relied upon. If greater accuracy is desired, the division may be extended further before commencing the contraction.

21. Divide 2704.1583 by 361.8901.
22. Divide 815.3796 by 21.55487.
23. Divide 14.289536 by 128.47.
24. Divide 2.81587 by 2643.
25. Divide 118.78 by 35.759.
26. What is the quotient of 6418 by 249.753?
27. What is the quotient of 297 by 36.8569.
28. Divide 2 by 375.814.
29. Divide 13 by .278113.
30. Divide 27 by 65.19428.

In Division of Circulating Decimals, we may adopt the following rule.

Make the repetends of the divisor and dividend similar and conterminous, and from the result, considered as whole numbers, subtract the finite part of each. Perform the division with the remainders as with whole numbers, and the true quotients will be obtained.

Divide $36.\dot{9}\dot{1}$ by $5.\dot{2}7\dot{3}$.

$$
\begin{array}{rl}
5.\dot{2}73273\dot{3}) & 36.\dot{9}19191\dot{1} \\
5 & 36 \\
\hline
5273268) & 36919155(7.001191 \\
\cdots\cdots & 36912876 \\
\hline
 & 6279 \\
 & 5273 \\
\hline
 & 1006 \\
 & 527 \\
\hline
 & 479 \\
 & 474 \\
\hline
 & 5 \\
 & 5
\end{array}
$$

The example is here solved by contracted decimal division. The exact fractional quotient is $7\frac{6279}{5273268}$ or $7\frac{23}{19316}$. The effect of subtracting the finite parts of the divisor and dividend is the same as reducing the two numbers to improper fractions, and dividing the numerators.

31. Divide $27.\dot{5}$ by $3.3\dot{4}1$.
32. Divide $8.9\dot{7}$ by $42.\dot{8}1\dot{5}$.
33. Divide 39 by $63.\dot{4}32\dot{8}$.
34. Divide $4.0\dot{2}\dot{1}$ by $65.\dot{3}7\dot{8}$
35. Divide $17.5\dot{8}32\dot{4}$ by 29.8.
36. Divide $41.31\dot{9}$ by $28.\dot{7}235\dot{8}$
37. Divide $121.6\dot{2}\dot{3}$ by $24.\dot{0}18\dot{4}$.
38. Divide $75.8\dot{1}\dot{4}$ by $364.\dot{2}\dot{5}$.
39. Divide $4.30\dot{9}\dot{7}$ by $18.6\dot{1}584\dot{3}$.
40. Divide $768.\dot{4}3\dot{2}$ by $29.5\dot{7}96\dot{1}$.
41. Divide $44.\dot{3}808\dot{1}$ by 27.85.

Continued Fractions, arise from the approximate valuation of fractions whose terms are large, and prime to each other. If, for example, we desire approximate values for the

fraction $\frac{89}{487}$, we may commence by dividing both terms of the fraction by the numerator, which gives us $\frac{1}{5\frac{42}{89}}$. Disregarding the $\frac{42}{89}$, we have $\frac{1}{5}$ for a first approximate value which is greater than the true value, because the approximate denominator is less than the true denominator. But as the denominator is between 5 and 6 the fraction is between $\frac{1}{5}$ and $\frac{1}{6}$.

If we desire greater accuracy, we may divide $\frac{42}{89}$ in the same manner as the first fraction, which gives us $\frac{1}{2\frac{5}{42}}$ for an approximate value of $\frac{42}{89}$, or

$$\cfrac{1}{5+\cfrac{1}{2\frac{5}{42}}},$$

for a second approximate value of the original fraction. Disregarding the $\frac{5}{42}$, the continued fraction becomes,

$$\frac{1}{5\frac{1}{2}},$$

or $\frac{2}{11}$, which is less than the true value, because the supposed denominator is greater than the true denominator. We therefore know that the fraction is between $\frac{1}{5}$ and $\frac{2}{11}$.

Still greater accuracy may be obtained by reducing $\frac{5}{42}$, which gives us

$$\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{8\frac{2}{5}}}}.$$

Rejecting the $\frac{2}{5}$, we have,

$$\cfrac{1}{5+\cfrac{1}{2\frac{1}{8}}}, \text{ or } \frac{1}{5\frac{8}{17}}, \text{ or } \frac{17}{93},$$

for a third approximate value, greater than the true value. The fraction is, therefore, between $\frac{17}{93}$ and $\frac{2}{11}$.

After one farther approximation, we should obtain the original fraction. In fractions whose terms are very large, as in the ratio of the diameter to the circumference of a circle, these approximate values are often very useful. They, moreover, have the advantage of admitting any re-

quired degree of accuracy, for the error in adopting any approximation, is always less than the difference between the fraction taken and the one following. Thus, in the present example, if we had adopted $\frac{1}{5}$ as the true value of $\frac{89}{487}$, the error would have been less than $\frac{1}{5}$ — $\frac{1}{6}$, or $\frac{1}{30}$.

For forming the successive approximations, we have the following

RULE.

Divide the greater term by the less, and the divisor by the remainder, &c., as in finding the greatest common measure.

Assume 1 *for the numerator, and the first quotient for the denominator of the first approximate value.*

Multiply the terms of this fraction by the second quotient, and add 1 *to the product of the denominator, for the second approximate value.*

For each succeeding approximation, multiply the terms of the last approximate fraction by the following quotient, and add the corresponding terms of the preceding fraction.

If the fraction given is improper, the reciprocals of the fractions thus obtained, will be the approximations desired.

Required less approximate values for the ratio of the circumference to the diameter of a circle, one approximate ratio being $\frac{314159}{100000}$.

```
100000)314159(3
       300000
        14159)100000(7
               99113
                  887)14159(15
                      887
                      5289
                      4435
                       854)887(1
                           854
                            33)854(25
                               66
                               194
                               165
                                29)33(1
                                   29
                                    4)29(7
                                      28
                                       1)4(4
                                         4
```

Equivalent Continued Fraction.

$$3+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{25+\cfrac{1}{1+\cfrac{1}{7\frac{1}{4}}}}}}}$$

For convenience, the fraction may be written,

$$3+\frac{1}{7+}\,\frac{1}{15+}\,\frac{1}{1+}\,\frac{1}{25+}\,\frac{1}{1+}\,\frac{1}{7+}\,\frac{1}{4}.$$

$\frac{1}{3}$ 1st. approximate value.

$\frac{1 \times 7}{3 \times 7 + 1} = \frac{7}{22}$ 2d. approximate value.

$\frac{7 \times 15 + 1}{22 \times 15 + 3} = \frac{106}{333}$ 3d. approximate value.

$\frac{106 \times 1 + 7}{333 \times 1 + 22} = \frac{113}{355}$ 4th. approximate value.

&c., &c.

The reciprocals of these values are,

$$3, \frac{22}{7}, \frac{333}{106}, \text{ and } \frac{355}{113}.$$

The second ratio is the one given by Archimedes. The fourth is that of Adrian Metius, and is *even more exact* than the ratio 3.14159, from which we have derived it.

42. Required the approximate values of $\frac{751}{2837}$.

43. Find the approximate values for $\frac{4999}{5763}$.

44. Find the approximate values for $\frac{34909}{72622}$.

45. What are the approximate values of .785398, which is nearly the ratio of the area of a circle, to that of its circumscribing square?

46. What are the approximate values of $\frac{37729}{74271}$?

47. Find approximate values for $\frac{42551}{13045}$.

48. Find approximate values for $\frac{44031}{31867}$.

In the EXTRACTION OF ROOTS, we may commence with any divisor, cutting off the right-hand figure at each step, as in contracted division. At whatever place this contraction is commenced, as many additional root figures will be obtained as are equal to the number of figures in the divisor *less* 1, but the last figure so obtained cannot always be relied upon. To illustrate this principle, we will extract the 5th root of 69.

In applying the general rule for obtaining any root, the pupil will frequently find great difficulty in determining the value of the figures in each column. To obviate this difficulty, he should be taught to supply the zeroes, when the first root figure is in the place of tens, hundreds, &c., and to observe the place of the decimal point in each product, until he becomes familiar with the process.

iven number, for a divisor. Divide, and reserve the
quotient, add the index of the root, plus 1,
times the index of the root

1	0	0			
	2	4			
	2	4	8	16	
	2	8	24	64	[illegible]
	4	12	32	80	4.63657
	2	12	48	27.8781	4.29498
	6	24	80	107.8781	34159
	2	16	12.927	32.0424	29334
	8	40	92.927	139.9205	4825
	2	3.09	13.881	3.246	4407
	10.3	43.09	106.808	143.166	418
	3	3.18	1.4	3.288	294
	10.6	46.27	108.2	146.454	124
			1.4	22	117
			109.6	146.67	7
				22	7
				146.89	

After obtaining the third trial divisor, we commence rejecting *one* figure from the trial divisor, *two* from the number at the foot of the preceding column, *three* from the third column, &c., and proceed in a similar way with each subsequent trial divisor, until the figures from the preceding columns are entirely cancelled. But in every instance, allowance must be made for the product of the figures rejected, as in simple contracted division.

49. Extract the square root of 287 ; of 5.

50. Extract the cube root of 11 ; of 25 ; of 693.

51. Extract the 4th root of 13 ; of 1.8 ; of 27.

52. Extract the 5th root of 797.9341.

53. Extract the 5th root of 1.0843.

54. Extract the cube root of 997641.285.

$\frac{1}{3}$ 1st. approximate

$\frac{1}{3 \times 7} = \frac{1}{7}$ 6819542.42.

$\frac{1}{3 \times 7 + 1} =$ of 27.91.

The SQUARE ROOT of any number may be expressed in the form of a continued fraction, after part of the root is found,—*by making each numerator equal to the remainder, and each denominator equal to twice the root found.* Thus in extracting the square root of 17, the first root figure is 4, and the remainder 1. Then the true root is 4 + the continued fraction

$$\frac{1}{8+}\frac{1}{8+}\frac{1}{8+}\frac{1}{8+}\frac{1}{8+} \text{ \&c.}$$

In like manner, the square root of 14, is

$$3 + \frac{5}{6+}\frac{5}{6+}\frac{5}{6+}\frac{5}{6+}, \text{ \&c.}$$

Reducing the fraction, we have, first,

$$\frac{5}{6\frac{5}{6}} = \tfrac{30}{41}, \text{ or } \tfrac{3}{4},$$

nearly, giving the first approximate root $3\frac{3}{4}$. Second,

$$\frac{5}{6\frac{30}{41}} = \tfrac{205}{276} \text{ or } \tfrac{17}{23}$$

nearly, giving a second approximate root $3\frac{17}{23}$. Third,

$$\frac{5}{6\frac{205}{276}} = \tfrac{1380}{1861} \text{ or } \tfrac{28}{31}$$

nearly, giving a third approximate root $3\frac{28}{31}$. This approximation is of use in affording convenient fractional expressions for those roots which are of most frequent occurrence. Thus, the diagonal of a square is to its side as $\sqrt{2}$ is to 1. By the rule just given, we obtain successively for approximate values of $\sqrt{2}$,

$$1 + \tfrac{1}{2}, \tfrac{2}{5}, \tfrac{5}{12}, \tfrac{12}{29}, \tfrac{29}{70}.$$

The last of these values, $1\frac{29}{70}$ or $\frac{99}{70}$, is a very convenient one.

The following is a general rule for the approximation of ANY ROOT desired.

RULE.

Call the first two figures of the root found in the usual way, the ASCERTAINED ROOT.

Involve the ascertained root to the given power, and multiply by the index of the root for a dividend.

Subtract the power of the ascertained root from the corresponding

periods of the given number, for a divisor. Divide, and reserve the quotient.

To 6 times the reserved quotient, add the index of the root, plus 1, for a second dividend.

To 6 times the reserved quotient, add 4 times the index of the root, subtract 2 from the sum, and multiply by the reserved quotient for a second divisor. Divide, add 1 to the quotient, and multiply by the ascertained root for the true root nearly. If greater accuracy is desired, repeat the process with the root thus found.

By this rule, the number of figures in surd roots; may generally be tripled at each operation.

The following is the application of the rule, in extracting the 5th root of 659901.

Ascertained root 14.

$14^5 =$	537824	given no.	659901
index	5	$14^5 =$	537824
dividend	2689120	1st divisor	122077

$2689120 \div 122077 = 22.02806$, reserved quotient.

$$
\begin{array}{rr}
\text{reserved quotient} & 22.02806 \\
 & 6 \\
\hline
 & 132.16836 \\
\text{index} + 1 & 6. \\
\hline
\text{second dividend} & 138.16836
\end{array}
$$

$$
\begin{array}{rr}
6 \times \text{by reserved quotient} = & 132.16836 \\
4 \times 5 - 2 = & 18. \\
\hline
 & 150.16836 \\
\text{Multiply by} & 22.02806 \\
\hline
\text{2nd divisor} & 3307.91764
\end{array}
$$

$138.16836 \div 3307.91764 = .041768$

$1.041768 \times 14 = 14.584752$, approximate root, correct to the fourth decimal place.

This contraction is of use in extracting the higher roots. Any root below the 10th may be obtained in the usual way, nearly as readily, and with much greater accuracy.

58. Find convenient fractional approximations to $\sqrt{3}$.

59. What are the approximate fractional values of $\sqrt{5}$?

60. Extract the 13th root of 1.08.

61. Extract the 17th root of 1.004.

62. Extract the 100th root of 1.07.

63. Extract the 45th root of 1.2.

64. Reduce $\sqrt{13}$ to a continued fraction.

65. What are the approximate values of the continued fraction which is equivalent to $\sqrt{27}$?

CHAPTER XXIV.

ANALYSIS.

All the operations of Arithmetic have for their object, the discovery of one or more unknown quantities; and the great difficulty in complicated questions, is to perceive the application of the simple rules which will lead to this discovery.

The examination of any question, in order to determine the relation of the different quantities to each other, is called Analysis. To keep the unknown terms more constantly in view, letters are frequently employed to represent them, and the work expressed in the statement of the question, is performed on these letters, as if their value was known, and we were proving the truth of the answer.

EXAMPLE FOR THE BOARD.

There is a fish whose head weighs 9 pounds; his tail weighs as much as his head and half his body; and his body weighs as much as his head and tail both. What is the weight of the fish?

Let x = the weight of the body.
$9 + \frac{x}{2}$ = the weight of the tail.
9 = the weight of the head.

$$x = 9 + \frac{x}{2} + 9$$
$$x = \frac{x}{2} + 18$$
$$\frac{x}{2} = 18$$
$x = 36$, the weight of the body.
$9 + \frac{x}{2} = 27$, the weight of the tail.
9, the weight of the head.
—
72, the weight of the fish.

In this example, if the weight of the body were known, the answer would be readily obtained. We therefore represent this weight, which is the *unknown quantity* by x. The tail weighing as much as the head and half the body, will be represented by

$$9 + \tfrac{1}{2} \text{ of } x.$$

But as $\frac{1}{2}$ of any number is the number divided by 2, $\frac{1}{2}$ of x will be $x \div 2$ or $\frac{x}{2}$.

The body weighing as much as the head and tail both,

$$x = 9, \text{(w't of head)} + 9 + \frac{x}{2} \text{ (w't of tail.)}$$

But

$$9 + 9 = 18, \text{therefore, } x = \frac{x}{2} + 18.$$

Now, if 18 added to the half of x gives x, 18 must itself be equal to $\frac{x}{2}$, and twice 18, or 36, to x. Having found the value of x, we easily obtain the remaining values.

The pupil may analyse the following examples, either with or without the aid of letters.

1. A father's age is 7 times that of his son, and the sum of their ages is 40. What is the age of each?

2. In a certain school there are 45 scholars, and there are twice as many boys as girls. Required the number of each?

3. A man performed a journey of 135 miles, going twice as far the second day as on the first, and three times as far the third day as on the second. How far did he travel each day?

4. A., B., and C., entered into partnership, contributing in the whole, \$4833. B. paid twice as much as A., and C. paid twice as much as A. and B. How much did each contribute?

5. In a certain school of 70 scholars, three times as many

study Arithmetic as study Latin, and twice as many learn to read, as study Arithmetic. How many are there in each study?

6. An estate of $7000 was so divided that the widow received $500 more than the daughter, and the son $1100 more than the widow. What was the share of each?

7. Divide the number 97 into four such parts that the second may be twice the first, the third 7 more than the second, and the fourth 18 more than the first.

8. A thief travels at the rate of 6 miles an hour, and after he has been absent $5\frac{1}{2}$ hours, a constable starts in pursuit, at the rate of 9 miles an hour. In what time will the thief be overtaken?

9. A man when he was married, was three times as old as his wife, but after they had lived together 15 years, he was only twice as old. How old was each at the time of marriage?

10. A farmer bought some cows and some calves for $461, giving $23 apiece for the cows, and $9 apiece for the calves, and there were twice as many calves as cows. How many were there of each?

11. At a certain election, the successful candidate had 163 votes more than his opponent, and the whole number of votes polled was 1125. How many did each receive?

12. What sum of money will yield $123.50 in 2 years, at 5 per cent. simple interest?

13. A gentleman distributed $1.95 among 3 beggars, giving the second 25 cents more than the first, and the third twice as much as the second. How much did each receive?

14. If from three times a certain number 17 be subtracted, the remainder will be 112. What is the number?

15. A's age is $2\frac{1}{2}$ times B's, and the sum of their ages is 49. Required the age of each.

EXAMPLE FOR THE BOARD.

A man being asked his age, replied, "If $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{3}{5}$

of my age and 19 years more be added to my age, the sum will be 3 times my age." What was his age?

Let $x =$ his age.

$$x + \frac{x}{2} + \frac{x}{3} + \frac{x}{4} + \frac{3x}{5} + 19 = 3x,$$

$$x + \frac{30x}{60} + \frac{20x}{60} + \frac{15x}{60} + \frac{36x}{60} + 19 = 3x,$$

$$\frac{161x}{60} + 19 = \frac{180x}{60}$$

$$19 = \frac{19x}{60}$$

$$1 = \frac{x}{60}$$

$$60 = x$$

After stating the question, we reduce all the fractions to a common denominator, and add them as in ordinary Addition of fractions. We then find that

$$\frac{161x}{60} + 19 = \frac{180x}{60}.$$

Therefore, 19 must be the difference between

$$\frac{161x}{60} \text{ and } \frac{180x}{60}, \text{ which is } \frac{19x}{60}.$$

If $19 = \frac{19x}{60}$, then $1 = \frac{x}{60}$, and $60 = \frac{60x}{60}$ or x.

16. A merchant owes two of his creditors \$1575, and he owes the second but $\frac{2}{3}$ as much as the first. What is the amount of each debt?

17. In a certain school $\frac{1}{2}$ the boys learn to read, $\frac{1}{5}$ learn to write, $\frac{1}{10}$ learn Algebra, $\frac{2}{15}$ learn drawing, and the remaining 4 study Latin. How many are there in the school?

18. One-third of a certain pole is painted green, and $\frac{2}{5}$ of it is painted white, the remainder, which is 8 feet, being in the ground. What is the length of the pole?

19. A man going to market, was met by another, who said: "Good morrow, neighbor, with your hundred geese."

He replied: "I have not a hundred; but if I had as many more, and half as many more, and two geese and a half, I should have a hundred." How many had he?

20. Three-sevenths of a certain number exceed $\frac{1}{5}$ of it by 24. What is the number?

21. A man bought 38 pounds of coffee and 95 pounds of sugar; he gave 2 cents per lb. more for the coffee than for the sugar, and the sugar cost twice as much as the coffee. What was the price of each per pound?

22. The age of a certain man when he was married, was to that of his wife, as 11 to 10; ten years afterwards the proportion was as 16 to 15. What was the age of each on the wedding day?

23. There are two numbers, such that if 21 be added to the first, the sum will be 5 times the second, and if 21 be added to the second, the sum will be 3 times the first. What are the numbers?

24. The sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{2}{7}$ of a certain number, is 575. Required the number.

25. The sum of three numbers is 96; the second is 2 more than the first, and the third is 4 more than the sum of the other two. What are the numbers?

26. Two stages are travelling towards each other, one at the rate of $5\frac{2}{7}$ miles an hour, the other $6\frac{1}{9}$ miles an hour. In what time will they meet, if they are now $38\frac{3}{4}$ miles apart?

27. Two men start from the same place and travel in opposite directions, one at the rate of $4\frac{1}{2}$ miles an hour, and the other $5\frac{3}{11}$ miles an hour. In what time will they be 100 miles apart?

28. A farmer hired a certain number of boys, and twice as many men, agreeing to pay each man 75 cents a day, and each boy 25 cents. The daily wages of the whole, amounted to $5.25. How many were there of each?

29. A gentleman owns a carriage that cost him 3 times as much as his horse, and both together cost $720. What was the price of each?

30. There is a number to which if $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{5}{6}$ of itself

be added, the sum will be $\frac{2}{3}$ of $8\frac{7}{9}$ less than 16. What is the number?

31. What number is that from which if we deduct $\frac{3}{7}$ of itself, and $\frac{2}{9}$ of the remainder, there will be 18 left?

32. If A. can do $\frac{1}{2}$ of a piece of work in 5 days, B. can do $\frac{1}{3}$ of it in 4 days, and C. can do $\frac{1}{8}$ of it in 2 days, in what time will they all do $\frac{2}{3}$ of it by working together?

33. Divide $87\frac{1}{2}$ into two such parts that one may be as much above $39\frac{1}{3}$ as the other is less than $39\frac{1}{3}$.

34. There are two men of equal ages, but if one was $5\frac{1}{2}$ years older, and the other $9\frac{1}{4}$ years younger, the former would be twice as old as the latter. Required their ages.

35. If $\frac{1}{2}$ and $\frac{1}{3}$ of a certain number be added, $\frac{8}{9}$ of the sum will be $1\frac{1}{8}$ greater than $\frac{2}{5}$ of $19\frac{1}{8}$. What is the number?

36. A man inquiring the distance to Boston, was told that if it was as far again and $\frac{2}{3}$ as far, and $\frac{1}{16}$ as far, it would be $3\frac{1}{4}$ miles less than $101\frac{1}{2}$ miles. What was the distance?

37. A laborer received \$1.50 for every day he worked, and lost 50 cents every day he was idle. He worked twice as many days as he was idle, and at the end of the time, he received \$42. How many days did he work?

38. $\frac{5}{9}$ and $\frac{2}{7}$ and $\frac{1}{6}$ and $\frac{3}{11}$ of a certain number, make 24. What is the number?

39. Of what number is $2\frac{5}{8}$, $9\frac{1}{3}$ less than $\frac{1}{2}$ of $\frac{3}{17}$?

40. There is a pole $\frac{4}{13}$ painted blue, $\frac{1}{2}$ painted white, $\frac{1}{17}$ painted red, and $2\frac{1}{2}$ feet unpainted. What is the length of the pole?

41. If a man can do $\frac{5}{11}$ of a piece of work in 3 days, and a boy can do $\frac{3}{7}$ of it in 5 days, how long will it take them both to do the whole?

42. A farmer hired an equal number of men and boys, giving each man 75 cents, and each boy $37\frac{1}{2}$ cents. How many were there of each, the wages of the whole being \$$10.12\frac{1}{2}$?

43. What is the height of a steeple that casts a shadow of $76\frac{2}{3}$ feet, when a pole 9 feet high casts a shadow of $5\frac{1}{2}$ feet?

44. The Dr. side of an account is $800, due May 16, and the Cr. side is $600, due May 1. When can the balance be paid without loss of interest to either party?

45. The Dr. side of an account is $800, due May 1, and the Cr. side is $600, due May 16. When can the balance be paid without loss of interest to either party?

46. A detachment of four regiments consisted of 4600 men. A.'s regiment contained 33 more than B.'s, 95 more than C.'s, and 200 more than D's. How many were there in each regiment?

47. B. is half as old again as A., C. is twice as old as B., and the sum of their ages is 55. Required the age of each.

48. B. is 2 years older than A., C.'s age is 4 years more than the sum of A's and B.'s, and D.'s age, which is 48, is equal to the sum of the other three. What is the age of each?

49. A teacher being asked the number of his pupils, replied, "If I had as many more, and $\frac{3}{4}$, and $\frac{5}{8}$, and $\frac{7}{20}$, and $\frac{5}{10}$ as many, I should have 31 less than 200." How many had he?

50. What sum of money will amount to $1500, in 15 years, at 5 per cent. simple interest?

51. At what rate per cent., simple interest, will $700 amount to $1300 in 11 years?

52. In what time will $1100 amount to $1750, at 6 per cent. simple interest?

53. A person, after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had 26\frac{2}{3}$ left. How much had he at first?

54. A cistern has three pipes; the first can fill it in $\frac{1}{2}$ an hour, the second can fill it in $\frac{1}{3}$ of an hour, and the third can empty it in an hour. In what time will the cistern be filled, if they all run together?

55. Five-sevenths of a certain number exceeds $\frac{1}{6}$ of $\frac{3}{11}$ of it by 46. What is the number?

56. A gentleman has a carriage worth $500, and two valuable horses. If the first horse be harnessed in the carriage, the horse and carriage will be worth 3 times as much as the second horse. But the second horse and carriage are

together worth 7 times as much as the first horse. What is the value of each ?

57. If $2\frac{1}{2}$ tons of merchandise can be carried $\frac{1}{4}$ of $96\frac{1}{3}$ miles for $3.75, what should be the freight of 1 cwt. for $7\frac{1}{3}$ miles less than $72\frac{5}{8}$ miles ?

58. A hare starts 5 rods before a greyhound, and runs at the rate of 12 miles an hour. After running 48 seconds, the hound starts in pursuit, and runs 20 miles an hour. In what time will the hare be overtaken ?

59. A bathing-tub that holds 147 gallons, is filled by a pipe that brings 14 gallons in 9 minutes, and emptied by a pipe that discharges 40 gallons in 31 minutes. Both pipes having been left open for 3 hours, it is required to find in what time the tub will be filled if the discharging pipe is closed ?

60. A spendthrift, after squandering $\frac{3}{7}$ of his fortune, and $\frac{2}{5}$ of the remainder, had $1500 left. How much had he at first ?

61. A younger brother received $1100, which was $\frac{5}{6}$ of his elder brother's fortune, and 5 times the share of the elder brother was twice as much as their father was worth. What was the father worth ?

62. Find two numbers in the proportion of 7 to 3, whose product is 189. (Let $7x$ and $3x$ be the numbers, then the product will be $21x^2$.)

63. There is a bin 4 feet high, that contains 299 cubic feet. What is the length of one side, the bottom being square ?

64. The length of a certain lot is to the breadth as 11 to 3, and the area is 2673 feet. What are the dimensions of the lot ?

65. The product of two numbers is 398, and the quotient of the greater by the less is 23. Required the numbers.

66. The length of a box is to the breadth as 4 to 3 ; and the depth to the breadth, as 2 to 3. What are the dimensions, the solid contents being 319144 cubic inches ?

67. Four men joined in a speculation, and the gain was so divided that A., B., and C., together, received $487 ; A.,

B., and D., \$494; B., C., and D., \$506; and A., C., and D., \$481. What was each man's share of the gain?

68. Three men traded in partnership. A. contributed \$1500, B. \$2250, and C. the remainder. The whole gain was \$2700, of which C. received \$1200. How much did C. contribute, and what did A. and B. gain?

69. A. and B. can do $\frac{11}{15}$ of a piece of work in 1 day; B. and C. can do $\frac{9}{10}$ of it; A. and C. can do $\frac{5}{6}$ of it in the same time. In what time will they all do it working together?

70. An estate of \$15000 is to be divided among three persons; A. is to receive \$$5\frac{1}{3}$ as often as B. receives \$$4\frac{1}{2}$, and B. is to receive \$$3\frac{7}{8}$ as often as D. receives \$$4\frac{1}{4}$. What is the share of each?

71. Three partners gained \$1230, of which A. received \$400, B. \$350, and C. \$480. A's stock was in trade 6 months; B's, 5 months; and C's stock, which was \$1800, was in trade 8 months. Required the stock of A. and B.

72. The difference between two numbers is 8, and the sum of their squares is 82. What are the numbers? (Let x = the less, and $x + 8$ = the greater. The equation will be $2x^2 + 16x + 64 = 82$. As 64 is added to $2x^2 + 16x$ to make 82, if 64 be subtracted from 82, the remainder must be equal to $2x^2 + 16x$. The value of x may then be obtained by the general rule for the extraction of roots.)

73. The difference of two numbers is 11, and their product is 20. Required the numbers.

74. The area of a certain field is 187 square rods, and the length exceeds the breadth by 6 rods. What are the dimensions?

75. There are two numbers whose difference is 4, and their product is to the sum of their squares, as 6 to 13. What are the numbers?

76. A farmer purchased a number of geese for £6 5*s.* He retained 5, and sold the remainder for 1*s.* 3*d.* apiece more than he paid, thus receiving what he paid for the whole. How many did he buy? (Let x = the number, then 1500*d.* $\div\ x$ will be the price paid for each, in pence.)

CHAPTER XXV.

MISCELLANEOUS PROBLEMS.

CHRONOLOGY.

According to the Julian Calendar or Old Style, the solar year was considered as being 365 days and 6 hours. The 6 hours in 4 years amounted to a day, therefore every fourth year was called a Leap Year, and consisted of 366 days.

But the true solar year is about 11 minutes less than the Julian year, and on this account, in 1582, it was found that Spring commenced 10 days later than at the establishment of the Julian Calendar. Pope Gregory the XIIIth, therefore, caused ten days to be taken out of the month of October in that year, and to prevent the recurrence of a similar variation, he ordered the centurial years should not be regarded as Leap Years, unless the number of centuries were divisible by 4.

This computation, which is called the Gregorian or New Style, was soon adopted in the greater part of Europe; but in England and America, the change was not made until 1752, when the error had amounted to eleven days. It was then ordered that the 3d of September should be called the 14th, and the Gregorian calendar adopted for the future. In Russia and Greece, the Old Style is still retained.

One of the first seven letters, A, B, C, D, E, F, G, is attached to every day in the year; thus, A is applied to Jan. 1st, 8th, 15th, &c.; B, to Jan. 2d, 9th, 16th, &c.; C, to Jan. 3d, 10th, 17th, &c. In this manner all days in any year which have the same letter, fall on the same day of the week. The dominical letter for any year is the letter that falls against all the Sundays. Thus, the 5th of January, 1845, will fall on Sunday, and the dominical letter will, therefore, be the 5th letter, or E. But in Leap Year there are two dominical letters, the first for January and February, the second for the remainder of the year. The following are the dominical letters for a few years to come; 1844, G,

F; 1845, E; 1846, D; 1847, C; 1848, B, A; 1849, G; 1850, F; 1851, E; 1852, D, C, &c.

PROBLEM I.

To find the Dominical Letter for any year, according to the Julian or OLD STYLE.

RULE.

To the given year add one-fourth of itself, plus 4, and divide the sum by 7. If there is no remainder, the dominical letter is G; if 1 remainder, F; and so on in inverse order. If the given year be Leap Year, the letter thus found will be the dominical letter for the last 10 months, and the next following letter, for the remainder of the year.

What was the dominical letter for A. D. 1531?

Given year	1531
one-fourth	382
	4
	7)1917
	273 + 6 remainder.

To the given year, we add one-fourth of itself, (rejecting the fraction,) and 4. Dividing this sum by 7, we have a remainder 6, which indicates that the dominical letter sought is the 6th from G, counting in retrograde order, which is A.

What were the dominical letters for A. D. 564?

564
141
4
7)709
101 + 2

The remainder 2, indicates that the dominical letter is the 2d from G, or E. But the year being leap year the dominical letter for January and February, will be the next following, or F. The two letters sought are therefore F, E.

If the given year were before the Christian era, the remainder would indicate the direct order of the letter. Thus, 1 denotes A; 2 denotes B; 5, E, &c.

1. What was the dominical letter for A. D. 769?
2. What were the dominical letters for A. D. 1492?
3. What were the dominical letters for A. D. 1620?
4. What was the dominical letter for A. D. 79?
5. What were the dominical letters for 752 B. C.?
6. What were the dominical letters for 2348 B. C.?
7. What was the dominical letter for 1821 B. C.?

PROBLEM II.

To find the dominical letter for any year, according to the Gregorian or NEW STYLE.

RULE.

Divide the centuries by 4, and take the remainder from 3. Add twice this remainder to $\frac{5}{4}$ of the odd years, and divide the sum by 7. If there is no remainder, the dominical letter is G; if 1 remainder, F, &c., as in the preceding rule.

What is the dominical letter for 1895?

$$\begin{array}{r} 4)\underline{18}\text{ cent.} \\ 4 + 2 \\ 3 - 2 = 1 \\ 2 \times 1 = 2 \\ \text{Odd years } 95 \\ \underline{23} \\ 7)\underline{120} \\ 17 + 1 \end{array}$$

Dividing 18 centuries by 4, there is 2 remainder. Taking this remainder from 3, we have a remainder of 1. Twice 1 added to 95 years plus $\frac{1}{4}$ of 95, (rejecting the fraction,) gives 120, which divided by 7 gives a remainder 1, indicating that the dominical letter is the 1st below G, which is F.

8. Find the dominical letter for 1833.
9. Find the dominical letters for 1856.
10. Find the dominical letters for 2040.
11. Find the dominical letter for 1911.
12. Find the dominical letter for 1799.
13. Find the dominical letters for 1876.
14. Find the dominical letter for 1921.

PROBLEM III.

To find the day of the week corresponding to any given day of the month.

RULE.

The dominical letter found by one of the preceding rules, will indicate the day on which the first Sunday in January will fall. The day of the week for the corresponding day of each succeeding month, may be found by the initials of the following couplet:

At Dover Dwells George Brown Esquire,
Good Captain French, And David Friar.

On what day of the week was the Declaration of Independence signed ?

The dominical letters for 1776 were G, F. Therefore the first Sunday in January was the 7th of the month. Then A representing the 7th Jan., D would represent the 7th Feb.; D the 7th March; G the 7th April; B the 7th May; E the 7th June; and G the 7th July. But 1776 being a Leap Year, the dominical letter after February is one day earlier in the month, and a day of the month which would otherwise be represented by G, will be represented by A or Sunday. The 7th July, therefore, came on Sunday, and the 4th on Thursday.

The initials O. S. denote the Old Style. In all cases not thus marked, the New Style is understood.

15. Washington was born on the 22d Feb. 1732. What was the day of the week?

16. The pilgrims landed at Plymouth, Dec. 11, 1620, O. S. What was the day of the week?

17. The Battle of Waterloo was fought June 18, 1815. Is it probable that a letter, purporting to have been written at the time, and dated Friday, June 18, is authentic?

18. On what day of the week was Oct. 11, 1492, O. S., the day that Columbus discovered America?

19. On what day of the week did Columbus set sail, Aug. 3, 1492, O. S.

20. On what day of the week will a note, at 90 days, dated Aug. 20, 1844, become due, allowing 4 days grace?

21. On what day of the week will a note, at 60 days, dated May 27, 1844, become due, allowing 3 days grace?

22. Washington died on the 14th day of the last month of the last year of the last century. What was the day of the week?

23. On what day of the week will be the 3d of April, 1896?

MENSURATION.

PROBLEM I.

To find the area of any surface bounded by four sides, the opposite sides being equal.

RULE.

Multiply one of the sides by the perpendicular let fall upon it, from the opposite side.

1. The length of an oblong rectangular field is 40 rods, and the breadth 16 rods. How many square rods does it contain? How many acres?

2. What are the contents of a four-sided field, whose opposite sides are equal, the length being 81 rods, and the distance between the longest sides, 13 rods?

3. The average length of Pennsylvania is about 300 miles, and the breadth 157 miles. What is the area?

4. What is the area of a rhombus, the side being 73.5 rods, and the breadth 61.25 rods?

5. How many square feet in a board that is $14\frac{3}{4}$ ft. long, and 11 inches wide?

6. How many square yards in a floor 14.3 ft. long, and $10\frac{1}{3}$ ft. wide?

PROBLEM II.

To find the area of a trapezoid, or figure of four sides, two of which are parallel.

RULE.

Multiply the sum of the two parallel sides by half the distance between them.

7. The two parallel sides of a trapezoid measure 11 and 15 inches respectively, and the height is 8 inches. What is the area?

8. What is the area of a field, two sides of which are parallel, and measure 72.5, and 89.25 rods, the distance between them being 39 rods?

9. What is the area of a trapezoid, one of the parallel sides measuring 96 rods, the other $63\frac{3}{4}$ rods, and the distance between them being 84.6 rods?

PROBLEM III.

To find the area of a triangle.

RULE.

Multiply one of the sides by one half of the perpendicular let fall from the opposite angle.

10. How many acres in a triangular meadow, one side measuring $127\frac{3}{5}$ rods, and the perpendicular 82.41 rods?

11. The base of a triangular lot is $96\frac{5}{8}$ rods, and the perpendicular distance from the opposite angle is 35 rods. What is the area?

12. What is the area of a triangle whose base is $11\frac{3}{7}$ inches, and perpendicular, 28.49 inches?

Any surface bounded by straight lines, may be divided into triangles, and the area of each triangle obtained. The sum of the several areas is the area of the whole surface.

PROBLEM IV.

To find the area of a circle.

RULE.

Multiply half the diameter by half the circumference, or multiply the square of the diameter by .785398.

13. The diameter of a circle is $36\frac{2}{5}$ feet. What is the area?

14. What is the area of a circle whose diameter is $9\frac{7}{8}$ miles?

PROBLEM V.

To find the area of an ellipse, the two diameters being given.

RULE.

Multiply the longer by the shorter diameter, and the product by .785398.

15. A house lot in the form of an ellipse has one diameter 110 feet, and the other 45 feet. What is the area?

16. What is the area of an ellipse, whose diameters are 25 and 17.5 feet?

PROBLEM VI.

To find the surface of a sphere.

RULE.

Multiply the square of the diameter by 3.1415926 ; *or, multiply the diameter by the circumference.*

17. What is the area of the earth's surface?

18. The circumference of a globe is 252 inches. What is the area?

19. The diameter of a globe is 11.5 inches. What is the area?

PROBLEM VII.

To find the area of the convex surface of a cylinder.

RULE.

Multiply the circumference of the base by the height of the cylinder.

20. The diameter of a cylindrical column is 6 feet, and the height 60 feet. What is the area of the convex surface?

21. What is the area of the whole surface of a cylinder, whose diameter is 7.5 feet, and height 49 feet?

PROBLEM VIII.

To find the solid contents of a cylinder.

RULE.

Multiply the area of the base by the height.

22. The diameter of a cylinder is 13 inches, and the height 69 inches. What are the solid contents?

PROBLEM IX.

To find the solid contents of any cylindrical body, whose sides taper uniformly,* as the trunk of a tree.

* Such a body is called the *frustrum of a cone.*

RULE.

Multiply together the diameters of the two extremities, and to the product add one-third of the square of the difference of the diameters. Multiply this sum by .785398, and the product will be the mean area between the two extremities. The mean area multiplied by the length, will give the solid contents.

23. What are the solid contents of a stick of timber, whose length is 50 feet, the diameter of the larger end 36 inches, and the diameter of the smaller end 30 inches?

24. What are the solid contents of a ship's mast, whose length is 35 feet, the diameter at the base 24 inches, and the smaller diameter 18 inches?

PROBLEM X.

To find the solid contents of a sphere.

RULE.

Multiply the cube of the diameter by .5236.

25. The diameter of a globe is $4\frac{2}{5}$ feet. What are the solid contents?

26. What are the solid contents of the earth, and what does it weigh, supposing the mean weight to be twice that of water?

PROBLEM XI.

To gauge, or find the dimensions of a cask.

RULE.

Find the diameter at the bung, the diameter at the head, and the length of the cask, all in inches. Subtract the head diameter from the bung diameter, and note the difference.

If the staves of the cask be much curved, multiply the difference by .7; if little curved, by .6; if of a medium curve, by .65; and if nearly or quite straight, by .55, and

add the product to the head diameter. The sum will be a mean diameter, by which the cask is reduced to a cylinder.

Multiply the square of the mean diameter by the length, and divide the product by 359 *for the contents in beer gallons, or by* 294 *for the contents in wine gallons.*

27. How many wine gallons will fill a cask whose bung diameter is 40 inches, the head diameter 30 inches, and the length 50 inches?

28. How many beer gallons will a cask contain, which measures 31 inches at the head, 33 inches at the bung, and 47 inches in length?

29. What are the contents in wine measure of a tub, whose inner diameter at the bottom is 29 inches, at the top 36 inches, and the height 30 inches? (The tub is a *frustrum of a cone*, and the solid contents are found by Problem IX.)

30. How many gallons will fill a churn, that is 18 inches in diameter at the bottom, 12 inches at the top, and 3 feet in height?

PROBLEM XII.

To find the carpenters' tonnage of a vessel.

RULE.

Multiply the breadth at the main beam, half the breadth, and the length, together. Divide the product by 95 *and the quotient is the tonnage.*

This is probably the best general rule for forming estimates; but no rule can be given that will produce a perfectly accurate result. The rule employed by government, in the collection of revenue, gives about $\frac{3}{5}$ of the tonnage thus obtained.

31. What is the tonnage of a vessel, whose length is 70 feet, and breadth 25 feet?

32. The length of a vessel is 163 feet, and the breadth 31 feet. Required the tonnage.

33. Find the tonnage of a vessel that is $113\frac{1}{2}$ ft. long, and $24\frac{3}{4}$ ft. wide.

34. What is the tonnage of a ship that is 116 feet long, and 31 feet wide?

NATURAL PHILOSOPHY.

PROBLEM I.

To find the specific gravity of a body.

RULE.

If the body is heavier than water, weigh it both in water and out of water, and the difference will be the weight lost in the water. Then, the weight lost in the water : the whole weight : : the specific gravity of water : the specific gravity of the body.*

But if the body is lighter than water, attach to it another body heavier than water, so that the two may sink together. Weigh the two together, and the heavier by itself, both in water and in the air, and find the loss of each in the water. Subtract the less loss from the greater, and say, the last remainder : the weight of the body in air : : the specific gravity of water : the specific gravity of the body.

1. A piece of gold weighed $36\frac{4}{7}$ dwt. in water, and $38\frac{6}{7}$ dwt. in the air. What was the specific gravity?

2. What is the specific gravity of a body that weighs $13\frac{1}{2}$ pounds in the air, and $9\frac{3}{4}$ pounds in water?

3. What is the weight of a block of oak, that contains $13\frac{2}{5}$ cubic feet, the specific gravity being .925?

PROBLEM II.

To find the distance at which bodies may be seen at sea, or on level ground, the height being known.

* The specific gravity of water is 1. A cubic foot of water weighs about 1000 oz., Av. Therefore the specific gravity of any body in thousandths, will represent the weight of a cubic foot in ounces.

RULE.

To the earth's diameter, (41815224 feet,) add the height of the eye, and multiply the sum by the height of the eye. The square root of the product is the distance at which an object ON THE SURFACE *of the earth or water can be seen.*

Work in the same way with the height of the object, and the sum of the two results is the distance at which the object may be seen.

How far may a mountain, that is $1\frac{1}{2}$ miles high, be seen from the mast-head of a ship, 50 feet above the surface of the water?

$\sqrt{(41815224 + 50) \times 50} = 45724$ ft. or $8\frac{2}{3}$ ms.
$\sqrt{(41815224 + 7920) \times 7920} = 575534$ ft. or 109 ms.
Ans. $117\frac{2}{3}$ ms.

4. How far can Bunker Hill Monument, which is 282 feet above the level of the sea, be seen from the deck of a vessel, the spectator's eye being 15 feet above the water?

5. How far may a mountain $2\frac{1}{2}$ miles high, be seen from the mast-head of a vessel, 40 feet above the water?

PROBLEM III.

To determine the distance of a gun, or a thunder cloud, from seeing the flash, and hearing the report.

RULE.

Multiply the number of seconds that elapse between the flash and the report by 1142, *for the distance in feet.*

6. Four and a half seconds after seeing the flash of a cannon, the report was heard. What was the distance?

7. What is the distance of an electrical cloud, if the thunder is heard in $2\frac{1}{2}$ seconds after the flash is seen?

PROBLEM IV.

To find the pressure of water against the banks of a stream or the dam of a pond.

RULE.

Multiply the area of the bank by one half the depth of the water, for the cubical contents of a column of water equivalent to the pressure.

8. The gate of a floom is 18 feet deep and 16 feet wide. What pressure does it sustain?

9. What amount of pressure is sustained by a bank whose area is 5694 feet, the average depth of water being 10.5 feet?

QUESTIONS FOR REVIEW.

EVERY PRINCIPLE INVOLVED IN THE FOLLOWING QUESTIONS SHOULD BE FULLY AND CLEARLY EXPLAINED BY THE PUPIL.

What is ARITHMETIC? What is a number? In how many ways are written numbers expressed? How many figures are employed for the purpose? What are they called? Why are they so called? What is a unit? What is an abstract number?—an applicate number? What is an integer?—a fraction? Of how many operations does Arithmetic consist? What are they called? What is the object of each?

What is NUMERATION? How many modes of numeration are now in use? Describe the Roman method? What is the leading principle of the Arabic method? How many characters are employed to represent numbers? What does zero represent? What is its use? What is the decimal point? How many *places* are embraced in a *period?* Repeat the numeration table for integers—for decimals.* How are decimals read? What is the effect of zeroes at the right of decimals? How may this be shown? Write and explain on the board, a number containing seven periods of integers, and twenty places of decimals. How many figures were embraced in a period, in the ancient English system of Numeration?

* The table may be continued to any extent. The denominations above duodecillions, to the twenty-second period, are: Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions.

What is ADDITION? How is the sign *plus* written, and what does it denote? How is the sign of equality written? How are numbers written in Addition? Propose an example in Addition, and perform it on the board, giving the reason for each step of the process. How may Addition be proved?

What is SUBTRACTION? What is the Minuend?—the Subtrahend?—the Remainder? What other terms are applied to the Remainder? What is the form of the sign *minus*, and its use? How are numbers written in Subtraction? If any figure of the Subtrahend exceeds the one above it, what may be done? Show that the true result may thus be obtained. How is Subtraction proved? Propose an example in Subtraction, and perform it on the board, giving the reason for each step.

What is MULTIPLICATION? What is the Multiplier?—the Multiplicand?—the Product? What are the factors of a number? What is a prime number?—a composite number? What is the sign of Multiplication? How is multiplication performed? How proved? How many decimals are pointed off in the product? Explain the reason for this. How may any number be multiplied by 10, 100, 1000, &c.? What other abbreviations may be adopted? How may the product be obtained in a single line? Propose and explain an example in Multiplication.

What is DIVISION? What is the Divisor?—the Dividend?—the Quotient?—the Remainder? What is a Fraction? What is the Numerator?—the Denominator? How may the remainder of any division be expressed in the form of a fraction? How is division performed? How many decimals are contained in the quotient? How is division proved? What abbreviations may be adopted? How is the true quotient figure obtained? What is a multiple?—a common multiple?—a sub-multiple? How is the least common multiple found? How may it be found by a table of prime factors? What is a common divisor, or common measure? How is the greatest common divisor found? How may it be found by a table of prime factors? Propose and explain an example in Division;—in finding the least common multiple;—the greatest common measure.

In how many different ways may FRACTIONS be regarded?

Show the application of each. What are the terms of a fraction? What is a proper fraction?—an improper fraction?—a mixed number?—a compound fraction?—a complex fraction? How may we reduce an improper fraction to a whole or mixed number?—a whole number to a fraction having any given denominator?—a mixed number to an improper fraction?—a compound fraction to a simple one?—a fraction to a decimal?—a decimal to a fraction?—a fraction to its lowest terms?—two or more fractions to a common denominator? Propose and explain an example of each reduction. Propose and explain examples in Addition, Subtraction, Multiplication, and Division of Fractions.

What are CIRCULATING DECIMALS, and whence do they arise? What is the repetend?—the finite part? How may we reduce infinite decimals to fractions? How may we divide by any number of 9's? How, when all but the units' figure are 9's? Give an example of each division, and explain the principle on which it depends.

What are COMPOUND NUMBERS? How are the operations on them performed? Propose and explain an example in reducing higher denominations to lower;—lower denominations to higher;—lower denominations to the fraction of a higher;—to the decimal of a higher. Propose and explain an example in Compound Addition;—Compound Subtraction;—Compound Multiplication;—Compound Division. What are Duodecimals? Perform examples in Multiplication and Division of Duodecimals, and explain each principle involved.

Prove an example in each of the simple rules, by CASTING OUT THE NINES. Show the application of the principle to errors of transposition.

How may business operations in which Compound Numbers are involved, be frequently abbreviated? Give an example? Propose examples of abbreviations in multiplying by any number of 9's;—in multiplying and dividing by 5;—by 25;—by 75;—by 125;—by 375;—by 625;—by 875;—in multiplying by any number within 12 of 100, 1000, &c.;—in squaring a number ending in 5;—in multiplying two numbers in which the tens are alike and the sum of the units is 10;—in finding the product of two numbers, one of

which is as much above, as the other is below, a certain number of tens;—in multiplication, when one figure of the multiplier is an aliquot part of some of the remaining figures.

What is PERCENTAGE? Give and explain an example of the application of percentage to Commission;—Insurance;—Taxes;—Stocks;—Gain and Loss;—Duties;—Simple Interest;—Compound Interest;—Discount. What is the distinction between Simple and Compound Interest? What is the Principal?—the Rate?—the Amount? What is the usual rate? What is the Bank Rule for computing interest?—the Decimal Rule? Explain the principles on which each is founded. Give an example of an Account Current. How do we find the rate, when the principal, interest, and time are given?—the time, when the principal, interest, and rate are given?—the principal,—the time, rate and interest being given?—the principal,—the time, rate and amount being given? What is the usual mode of computing discount?

Propose and explain examples in EQUATION OF PAYMENTS, and AVERAGE.

What is RATIO? What is a proportion? What are the terms of a proportion called? What are the antecedents?—The consequents?—The extremes?—The means? What is said of the product in a proportion? Prove the fact. When one of the extremes, and the two means are given, how may the other extreme be found? How may the antecedents and consequents be diminished? State the RULE OF THREE, and solve a question by it. Explain the distinction into multipliers and divisors;—into cause and effect. What simple rule is founded on these distinctions? How is the statement made when the terms are fractional? Why? What is Arbitration of Exchange? What is the Chain Rule? Show the connection of cause and effect, in the Chain Rule. Propose and explain examples in Proportion, and Arbitration of Exchange.

What is the object of the rule of FELLOWSHIP? Propose and explain examples to which the rule is applicable.

What is ALLIGATION? How may any desired mixture be made, when there is no limit?—when the whole quantity is limited?—when one or more of the ingredients is limited?

What does PERMUTATION show? Give an example, and explain the rule. What is COMBINATION? Propose an example, and explain the rule by which it is solved.

What is INVOLUTION? What is the product obtained by Involution called? What is the root?—the 2d power?—the 5th power?—the index, or exponent? What other names are given to the 2d and 3d powers?—Why? What is the effect of adding the exponents of two powers of the same number?—of subtracting the less exponent from the greater? —of multiplying the exponent by any number? Explain each case.

What is EVOLUTION? What is a radical sign?—a fractional exponent? What is indicated by the numerator of a fractional exponent?—by the denominator? What is a rational number?—a surd? What is meant by the extraction of the square root? Explain the mode in which it is done, and perform an example. To what is the square of any number equivalent? Prove that this is true by an example. To what are the areas of similar figures proportional? What relation exists between the sides of a right-angled triangle? What is a mean proportional? How is it found? What is the mean proportional between 3 and 4? Mention some of the properties of square numbers.

What is the CUBE ROOT of a number? How may we determine the number of figures that any cube root will contain? Explain by blocks, the formation of cubes, and apply the principle to the extraction of cube roots. How may the trial divisors, after the first, be readily found? Mention some of the properties of cubes.

When the exponent of a power can be resolved into factors, how may the root be extracted? Show the application of this rule, by extracting the 8th root of 13. Apply the general rule by extracting the 5th root of 101621504799.

What is ARITHMETICAL PROGRESSION? By what other name is it called? What are the extremes?—the means? —the common difference? What is an ascending series? —a descending series? What is required, in order to determine an equi-different series? When one extreme, the common difference, and the number of terms are given, how may the other extreme, and the sum of all the terms be

found? Illustrate the rule on the board. How is the common difference found, the extremes and number of terms being given? How is the rule deduced? The extremes and common difference being given, how do we find the number of terms? Why does this process give us the number of terms?

What is GEOMETRICAL PROGRESSION? By what other names is it called? What is the ratio? Propose examples, and explain on the board, the process for finding one of the extremes, when the other extreme, the ratio, and number of terms are given;—the sum of the terms,—the ratio, one extreme, and number of terms being given;—the ratio, when the extremes and number of terms are given. How may we insert any number of mean proportionals between two given numbers? Explain the analogy between Arithmetical and Geometrical Progression.

What is HARMONICAL PROPORTION?—Harmonical Progression? Why are they so called? Two terms of a harmonical progression being given, how are the remaining terms found? How may we insert any number of harmonical means between two given numbers?

What is an ANNUITY?—an annuity certain?—an annuity contingent?—an annuity in possession?—an annuity in reversion? How do we find the amount of an annuity in arrears?—the present worth of an annuity certain?—the present worth of a perpetual annuity?—of an annuity in reversion?

What is the commercial signification of EXCHANGE? What is a Bill of Exchange? Illustrate the operation of a Bill of Exchange, in the payment of debts. Is the bill generally transmitted directly? Who is the drawer?—the drawee?—the payee?—the acceptor?—the holder?—the indorsee? What two kinds of indorsement are there, and what is the distinction between them? How is a bill accepted? If payable after sight, what should be done on its acceptance? What is the true par of exchange? What is the peculiarity in the nominal par with England? What is the course of exchange, and by what is it caused? What is the limit to the premium on the *true* par of exchange? What is a protest? What is meant by Domestic Exchange? What term

is applied to Domestic Bills of Exchange? State the value of the old currencies of different sections of the Union.

When are numbers said to be PRIME TO EACH OTHER? How may we tell by inspection, whether a number is divisible by 2, 3, 4, 5, 6, 9, 10, 11, or 25? How may we determine whether a given number is a prime number? How is a number resolved into its prime factors? How do we discover all the divisors of a number? What is the rule for determining the number of divisors? How may the resolution into prime factors be applied in the reduction of fractions to their lowest terms?

What is the cause of NUMERICAL APPROXIMATIONS? Propose and explain examples in Addition and Subtraction of Circulating Decimals. In contracted multiplication and division;—in division of circulating decimals. What is a continued fraction? Explain the mode of finding approximate values for a continued fraction? How may any square root be resolved into a continued fraction? How may the extraction of roots of the higher powers be contracted?

What is the JULIAN YEAR? Explain the distinction between the Old and New Style? What is the Dominical Letter? How is it found for any Julian Year?—for any Gregorian Year? How may we find the day of the week, corresponding to any given day of the month in any year proposed?

How may we find the AREA OF A TRIANGLE?—of a rectangle?—of a trapezoid?—of any plain surface bounded by straight lines?—of a circle?—of an ellipse?—the surface of a sphere?—of a cylinder? What is the rule for finding the solid contents of a cylinder?—of the frustrum of a cone?—of a sphere? How do we find the dimensions of a cask?—the carpenters' tonnage of a vessel?

How do we find the SPECIFIC GRAVITY of a body?—the distance at which objects may be seen, at sea or on level ground?—the distance of a gun or thunder cloud?—the pressure of water against a bank?

Repeat the Table of AVOIRDUPOIS WEIGHT;—Troy Weight;—Apothecaries' Measure;—Dry Measure;—Liquid Measure;—Long Measure;—Time Measure;—Square Measure;—Cubic Measure;—Cloth Measure;—Chain Measure.

MISCELLANEOUS EXAMPLES.

1. If the multiplicand is 7, and the product 2, what is the multiplier?

2. The dividend is 1, and the quotient 8; what is the divisor?

3. The dividend is $6\frac{1}{2}$, and the quotient $18\frac{2}{3}$; what is the divisor?

4. The sum of two numbers is $7\frac{1}{3}$, and one of the numbers is 4.759; what is the other?

5. The difference of two numbers is $13\frac{3}{7}$, and the greater number is 29.43; what is the less?

6. The sum of two numbers is $7\frac{5}{19}$, and their difference is $6\frac{3}{31}$; what are the numbers?

7. What number must be subtractea from $39\frac{5}{9}$ to leave $17\frac{2}{25}$?

8. What number must be added to $23\frac{5}{11}$ to make 47.432?

9. What number must be multiplied by $3\frac{7}{8}$, and the product divided by 7.365 to give $8\frac{3}{4}$ as a quotient?

10. What is the difference between $\frac{3}{46}$ and $\frac{4.6}{8}$ of 7 T. 15 cwt. 3 qr.?

11. What is the difference between 28 miles, and 27 m. 7 fur. 39 r. 5 yd. 2 ft. 11.9 in.?

12. Reduce $\frac{1}{7}$ of 9 m. 7 fur. 39 r. 5 yd. 2 ft., to inches.

13. If from a purse containing £35 7s. 11d., I pay to each of 15 laborers, £1 9s. $8\frac{3}{4}$d., how much will be left?

14. Find the sum, the difference, and the product of 874.91, and $42\frac{7}{13}$.

15. If 29 men can build $47\frac{1}{2}$ rods of wall in $9\frac{1}{2}$ days, how much can 15 men build in $24\frac{3}{4}$ days?

16. A grocer sold 17 cwt. 3 qr. 17 lb. of sugar, at $6\frac{3}{4}$ cents a pound, receiving in exchange 29 barrels of flour at \$$4\frac{7}{8}$ per barrel, and the balance in money. How much money did he receive?

17. From $\frac{5}{7}$ of 3 T. 17 cwt., subtract $\frac{4}{13}$ of 7 T. 3 cwt. 1 qr. 18 lb.

18. What will be the freight of $17\frac{3}{5}$ cwt. for $89\frac{5}{7}$ miles, if \$7.63 be paid for carrying $11\frac{3}{13}$ tons, $9\frac{1}{3}$ miles?

19. How many raisins at $8\frac{1}{2}$ cents a pound, must be given in exchange for 163 gal. 2 qt. 3 gi. of wine, at \$1.12$\frac{1}{2}$ per gallon?

20. Bought 16 cwt. 3 qr. 16 lb. of rice, at \$4.00 per cwt. and 9 cwt. 2 qr. 5 lb. of pearl barley, at \$4.37$\frac{1}{2}$ per cwt. How much would be gained on the whole, by selling each at $4\frac{3}{4}$ cents a pound?

21. If $63\frac{5}{8}$ yd. of broadcloth cost \$255, at what price must it be sold per yd. in order to gain \$25.50?

22. A hogshead of sugar at \$7.00 per cwt. cost \$43.75. What did it weigh?

23. How much shalloon that is $\frac{3}{4}$ yd. wide will line $14\frac{5}{8}$ yards of cloth that is $1\frac{3}{4}$ yd. wide?

24. What is the price of 16 boxes of raisins, each holding $9\frac{2}{5}$ lb. at $9\frac{1}{4}$ cents per lb.?

25. How much money that is 9 per cent. below par, will pay a debt of \$187.50?

26. If 9 men mow 10 acres of grass in a day, how much will 11 men mow in $2\frac{1}{2}$ days.

27. In what time will \$150 gain \$6.37$\frac{1}{2}$ at 6 per cent. simple interest?

28. When molasses is $31\frac{1}{4}$ cents a gallon, how many hogsheads, each holding 97 gal. 3 qt. can I buy for \$366.56$\frac{1}{4}$?

29. What will be the price of 7 bales of sheeting, each bale containing 9 pieces, and each piece measuring $30\frac{3}{4}$ yd., if 26 yd. cost. \$2.92$\frac{1}{2}$?

30. Bought $39\frac{1}{2}$ bushels of potatoes for \$12.87$\frac{1}{2}$. At what price per bushel must they be sold, in order to gain 15 per cent.?

31. What is the interest of \$9431 for 3 yr. 7 mo. 13 dy. at 7 per cent.?

32. How much may a man spend per day, whose income is \$500 a year, after deducting $\frac{6}{10}$ per cent. for taxes?

33. If 16 gallons of cider can be bought for \$2.75, how much may be bought for \$63$\frac{4}{5}$?

34. How many pieces of merino that is worth \$$\frac{5}{8}$ per yd. can be bought for \$71.25, there being 28$\frac{1}{2}$ yards in a piece?

35. Bought 7 hogsheads of sugar at \$6.75 per cwt. To what did it amount, the grŏss weight being 6 cwt. 3 qr. 19 lb., tare 25 lb., per hogshead?

36. What will be the price of 13 bags of cotton, each weighing 5 cwt. 1 qr. 11 lb., at 11$\frac{1}{2}$ cents per lb.?

37. If $\frac{1}{2}$ of $\frac{3}{4}$ of 5$\frac{1}{3}$ yd. of cloth cost \$10$\frac{5}{8}$, what will 17.63 yards cost?

38. At £1 11s. per yd. what will be the price of 3 qr. 3 na. of cloth?

39. A. lent B. \$685 for 3$\frac{1}{2}$ months; what sum ought A. to receive in return, to use 7$\frac{2}{3}$ months?

40. How much coffee at 8$\frac{1}{4}$ cents a pound, should be given in exchange for $\frac{3}{11}$ of 2 cwt. 2 qr. 16 lb. of tea, at 75 cents a pound?

41. If \$63.87$\frac{1}{2}$ will purchase 3$\frac{2}{5}$ cwt. of butter, how much must I give for 7 cwt. 1 qr. 13$\frac{1}{2}$ lb.?

42. A bankrupt can pay but 63$\frac{1}{3}$ per cent. of his debts. How much will be received by a creditor for \$765.62$\frac{1}{2}$?

43. If $\frac{3}{7}$ of $\frac{5}{12}$ of a ship be sold for \$2150, what is the value of $\frac{2}{3}$ of $\frac{8}{9}$ of the remainder?

44. What is the value of 87 barrels of oil, each containing 85$\frac{7}{8}$ gal., at 9$\frac{3}{4}$ cents per pint?

45. If a laborer receives \$1.50 for every working day, how much can he spend each year, and save \$125?

46. When taxes on all real and personal property are rated at 3 mills on every dollar, what must be paid by a man who is worth \$7650, and who pays for 3 polls at \$1.50 each?

47. How much hay will 63 horses eat in 21$\frac{1}{3}$ weeks, if 19$\frac{3}{5}$ cwt. will keep 4 horses 6$\frac{1}{2}$ weeks?

48. A garrison of 500 men has provisions for 6 weeks, but finding that they will be obliged to remain 15 weeks

without an additional supply, how many men must be sent away, that the provisions may not be exhausted?

49. What part of 17A. 3R. 29r. are 11A. 2R.?

50. Reduce $16\frac{3}{5}$, 14.7, $\frac{11}{13}$, $5\frac{1}{4}$, and $\frac{29}{52}$, to a common denominator.

51. If a staff 3 ft. 3 in. high cast a shadow of 2 ft. 7 in., how long a shadow will be cast by a steeple that is $59\frac{5}{8}$ yds. high?

52. Bought 38 pieces of Denims, each piece containing $29\frac{3}{4}$ yd., for $162.50. At what price must I sell $16\frac{1}{4}$ yd. in order to gain 17 per cent. on the prime cost?

53. How much cotton, at $16\frac{1}{2}$ cents a pound, can be bought for $\$512\frac{1}{8}$?

54. A grocer bought 7 cwt. 3 qr. 13 lb. of coffee for $70.48. What per cent. did he gain by selling the whole at 8 cents a pound?

55. Sold 39 cwt. 2 qr. 17 lb. of coffee, for $7.50 per cwt., thereby losing $37.50. What did it cost per pound?

56. What is the value of an estate, if $\frac{5}{17}$ of $\frac{3}{20}$ of it is worth $2897?

57. A man can perform a journey of 263 miles in 9 days, by travelling $7\frac{1}{2}$ hours a day. In what time can he perform a journey of $571\frac{3}{4}$ miles, by travelling $6\frac{3}{8}$ hours a day?

58. At what price per yd. must I sell 765 yd. of calico, in order to gain $15.30, the prime cost being $101.25?

59. In what time will $487.50 at 5 per cent., gain the same interest as $943.25, in 7 months at 6 per cent.?

60. If 15 men in $21\frac{1}{2}$ days mow 301 acres of grass, how much will 84 men mow in $7\frac{3}{4}$ days?

61. How much indigo at $1.75 per pound, can be bought for £15 9*s.*, when exchange is $9\frac{1}{2}$ per cent. above par?

62. In what time will $638 gain $95, at $4\frac{1}{2}$ per cent. per annum?

63. The estate of a bankrupt is but $7350, and his debts amount to $28495. How much can he pay on a debt of $1763.25?

64. Two pieces of cloth, each containing $31\frac{3}{8}$ yd., cost

$311, but one being of better quality cost $17.75 more than the other. What was the price of each per yard?

65. How much merchandise can be carried 75 miles for $15.75, if the carriage of 7 cwt. 3 qr. is $2.25, for 18 miles?

66. If 3 men in 16 days of 11 hours, can dig a ditch 18 rods long, 3 ft. wide, and 2 ft. deep, what length of ditch, that is 6 ft. wide, and $3\frac{1}{2}$ ft. deep, can be dug by 18 men in 9 days of 8 hours?

67. If the 5 cent loaf weigh 1 lb. 3 oz., when flour is $4.75 per barrel, what should the 6 cent loaf weigh when 19 barrels of flour can be bought for $85.50?

68. By selling tea at $87\frac{1}{2}$ cents a pound, a grocer gains 22 per cent. What did he give for 1 cwt. 1 qr. $11\frac{3}{4}$ lb?

69. At what rate per cent. will $850 amount to $1430 in 11 Y. 7 mo. 19 dy.?

70. If 118 men in 4 days of $9\frac{1}{2}$ hours, dig a trench 120 ft. long, $5\frac{1}{2}$ ft. wide, and $2\frac{3}{4}$ ft. deep, what will be the width of a trench 211 ft. long, and 3 ft. deep, that 56 men will dig in 17 days of 8 hours?

71. Three men traded in company. A contributed $4500, B. $5300, and C. $5000; at the end of 6 mo. A. put in $2100 more; at the end of 10 mo., B. took out $1700; at the end of 4 mo. C. took out $975, but at the end of 11 mo. he again put in $1300. At the end of $17\frac{1}{2}$ months, they had gained $3750. What was the share of each?

72. A board that contains 15.41 square feet, is 1 3′ wide. What is the length?

73. A grocer gains 25 per cent., by selling tea at 83 cents per pound. How much would he gain per cent. by selling 1 cwt. 1 qr. 17.9 lb. for $125?

74. If 73 men can do a piece of work in 63 days, how many men must be added, to complete it in 15.75 days?

75. A bankrupt pays at one time $13\frac{1}{4}$ per cent. of his debts, at another, $7\frac{1}{2}$ per cent., at another, .095, and at another, $19\frac{3}{5}$ per cent. How much was received in the whole, on a debt of $875?

76. A butcher sold 763 hams at $5\frac{3}{4}$ cents per lb., the

average weight being $12\frac{3}{4}$ lb. How much rice, at \$4.25 per cwt., ought he to receive in exchange?

77. If $18\frac{3}{4}$ per cent. is lost by selling shoes at 75 cents a pair, at what price should they be sold to gain $12\frac{1}{2}$ per cent.?

78. An estate of \$7000 was to be divided among 4 legatees, in the proportions of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{3}{11}$. Required the share of each?

79. A factor purchased 13 bales of cotton, each containing 3 cwt. 2 qr. 7 lb., at $10\frac{1}{2}$ cents per lb. What was his commission, at $2\frac{7}{8}$ per cent.?

80. Sold 7 cwt. 1 qr. of sugar for \$51.359, the prime cost being $5\frac{3}{4}$ cents per lb. At what price per lb. must I sell coffee, to gain the same rate per cent., the cost of 2 cwt. 3 qr. 8 lb. being \$30.02?

81. Find the sum of £$1\frac{8}{13}$, £$7\frac{4}{9}$, $\frac{184}{11}$s., and $\frac{23098}{17}$d.

82. Four persons trade in company. A contributes $\frac{1}{4}$ of the stock, B. $\frac{6}{7}$ as much as A., C. $\frac{1}{2}$ as much as A., *plus* $\frac{3}{8}$ as much as B., and D. the remainder. It is required to divide the gain, which is \$763.50, equitably among them.

83. How much tea at 50 cents, $62\frac{1}{2}$ cents, 75 cents, and 90 cents a pound, should be mixed with 13 lb. at 70 cents, 15 lb. at 80 cents, and 12 lb. at \$1.00, to make a mixture worth 75 cents a pound?

84. How many changes could be rung on 15 bells?

85. In what time could a man walk $17\frac{5}{9}$ miles, by taking 11 steps in 7 seconds, and going 2.7 ft. at each step?

86. By travelling 7 miles an hour, for $11\frac{1}{2}$ hours a day, a journey can be performed in $13\frac{3}{4}$ days. How many days of $10\frac{3}{4}$ hours, will be required to perform the same journey, travelling $6\frac{1}{2}$ miles an hour?

87. If $7\frac{1}{3}$ times $7\frac{1}{3}$ lb. of chocolate, cost $10\frac{7}{8} + 1\frac{7}{8}$ dollars, how much will $\frac{1}{3}$ of $\frac{7}{9}$ of $58\frac{6}{13}$ lb. cost?

88. What is the length of the longest straight line that can be taken in a room, whose length is 16 ft., breadth 10 ft., and height $9\frac{1}{2}$ ft.?

89. At £1 5s. 3d. per yard, what will be the price of $17\frac{7}{8}$ yards of broadcloth?

90. If the earth contains twice as much water as land, how many tea-spoonfuls of water are there on the globe?

91. How many bricks, each 8 in. long, 4 in. wide, and 2 in. thick, are in a pile 37.3 ft. long, 9.3 ft. wide, and $8\frac{4}{5}$ ft. high?

92. At what time between 10 and 11 o'clock, will the hour and minute hands be exactly together?

93. What number, multiplied by $86\frac{7}{8}$, will give $1.3\dot{1}\dot{4}$?

94. What is the interest of \$1794, for 5 yr. 9 mo. 13 dy. at $4\frac{3}{4}$ per cent.?

95. When exchange at St. Louis, on New Orleans, is at 3 per cent. discount, exchange at New Orleans on Philadelphia, at 11 per cent. premium, exchange at Philadelphia on New York, at 5 per cent. premium, how much St. Louis money must be remitted by way of New Orleans and Philadelphia, to pay a bill of \$6370 in New York?

96. When the bills of the United States Bank were selling at a discount of $13\frac{1}{2}$ per cent., how much United States Bank money was required to pay a debt of \$641.50?

97. After cutting $\frac{2}{7}$ of $16\frac{1}{3}$ yards from a piece of cloth which originally contained 29 yd. 3 qr. 2 na., I wish to know at what price per yard the remainder must be sold, to bring \$117.95?

98. How many bricks, each $8\frac{1}{2}$ in. long, $4\frac{1}{4}$ in. wide, and $2\frac{1}{8}$ in. thick, will build a wall 183 ft. long, $3\frac{1}{2}$ ft. thick, and $6\frac{1}{3}$ ft. high?

99. What is the present worth of \$17941, due in 3 yr. 5 mo. 23 dy. at 6 per cent.?

100. What is the discount on a note at 90 days for \$753.62$\frac{1}{2}$, at 6 per cent.?

101. A man gave away $\frac{3}{11}$ of his farm, and sold $\frac{5}{8}$ of $\frac{3}{7}$ of the remainder for \$1384. What was the whole farm worth at that rate?

102. $28.\dot{0}9\dot{4}$ is $\frac{3}{7}$ of $\frac{2}{13}$ of what number?

103. How much wine at 75 cents, 83 cents, 90 cents, and

95 cents a gallon, must be mixed with 14 gallons of water, to make a mixture worth 85 cents a gallon?

104. A man paid \$4500 for building a house, on which he is obliged to pay the following annual expenses; for taxes \$19.75; ground rent \$75; collecting the rent \$9.00; repairs \$75. What per cent. does he receive on his investment, the house renting for \$350?

105. What part of $\frac{4}{7}$ of $\frac{10}{9}$ of 11 m. 7 fur. 16 r., is $\frac{2}{3}$ of $\frac{5}{7}$ of 5 times 4 fur. 11 r. 4 yd.?

106. A wall is to be built by five men. A. can build $\frac{2}{3}$ of it in 12 days; B. can build $\frac{3}{4}$ of it in 18 days; C. can build $\frac{1}{6}$ of it in 4 days; D. can build $\frac{1}{2}$ of it in 9 days; and E. can build $\frac{5}{9}$ of it in 20 days; in what time will it be completed?

107. A draper by selling a piece of cloth for \$60 lost $\frac{1}{4}$ of $\frac{2}{3}$ of $53\frac{1}{3}$ per cent. on the prime cost. What did the cloth cost him?

108. A. B. and C. by working together, can do a piece of work in 4 days. A. could do it alone in 15 days, and B. in 10 days; in what time could C. do it?

109. A man being asked his age, replied: "If $\frac{1}{2}$ of $\frac{7}{11}$ of $1\frac{1}{4}$, $\frac{1}{3}$ of $\frac{6}{4}$, $\frac{2}{7}$ of $\frac{3}{3}$ of $\frac{7}{5}$ of $\frac{1}{2}$, and $\frac{1}{3}$ of my age be added to my age, the sum will be 137." What was his age?

110. If 9999999 barrels of flour cost \$47499995.25, what is the cost of 1 barrel?

111. $83\frac{7}{8}$ is $\frac{5}{9}$ of $\frac{3}{8}$ of what number?

112. How much will be received by discounting a 60 days' note for \$5130, at 6 per cent.?

113. What number, diminished by $\frac{2}{5}$ and $\frac{7}{9}$ of itself, gives $18\frac{1}{3}$?

114. How many grains of wheat would reach around the globe, if 3 grains occupy an inch?

115. What is the difference between the equitable, and the bank discount of \$4380, for 7 months, at 6 per cent.?

116. A legacy of \$17000 is to be divided among four persons. A. is to receive $\frac{1}{3}$, B. $\frac{1}{5}$ of the remainder, C. $\frac{1}{2}$ of

as much as A. and B., and D. the rest. What is the share of each?

117. What number is that, which being divided by $\frac{3}{11}$ of .000841, gives 76.9?

118. How many drops of water in $\frac{5}{73}$ of 13 Cong. 3 oz. 11 f℥ 2 fʒ?

119. Multiply 8450.301 by 9999999.

120. How much was due on the following note, May 15, 1844, interest at 7 per cent.?

$975.50 New York, June 29, 1840.

For value received, we promise to pay William S. Harris, or order, Nine Hundred and Seventy-Five and $\frac{50}{100}$ Dollars, in four months, with interest afterwards.

WILSON & MANLY.

Endorsements. Feb. 3, 1841, received $13.50. July 7, 1841, received $17.25, Sept. 4, 1841, received $59.75. Jan. 13, 1842, received $12.00. Oct. 21, 1842, received $250. May 30, 1843. received $25.00. Nov. 7, 1843, received $17.00. Jan. 23, 1844, received $318.

121. Add $\frac{1}{5}$ of $\frac{3}{7}$ of .18 of $5\frac{2}{3}$ gal., 17 gal. 3 qt. 1 pt., $96\frac{2}{3}$ gal., and $5\frac{3}{11} \times \frac{2}{7}$ of $\frac{1}{3}$ qt.;—subtract from the sum $\frac{9}{11}$ of 2 gals. 3qt.;—multiply the remainder by 75.03;—divide the product by .00125; and reduce the quotient to gills.

122. Find all the divisors of 314600.

123. What is the weight of a mass of lead that is 4 in. long, 3 in. wide, and 2 in. thick, its specific gravity being 11.351?

124. A housekeeper has a teacup, a table spoon, a teaspoon, a pair of scales, and a pail of water, but she has lost her set of weights. How can she weigh 16 lb. 11 oz. 13 dr. with these utensils?

124. Multiply in one line, 7584 by 721.

125. The sum of two numbers is $64.\dot{2}\dot{7}$, and their difference is $18\frac{9}{11}$. Required the numbers?

126. Reduce $35\frac{8}{17}$ miles to inches.

127. An accountant in comparing his books, discovers an error of $495. How did the error probably arise?

128. How far will a locomotive run in 5 h. 29 m. 47 sec., at the rate of $19\frac{3}{8}$ miles an hour?

129. The product of three factors is $16\frac{1}{3}$, and two of the factors are $15\frac{1}{2}$ and $7\frac{3}{4}$. What is the third?

130. $8\frac{2}{3} \times 3\frac{4}{5} \div 96\frac{1}{7} = ?$

131. What is the value of 187 francs 79 centimes, at $18\frac{1}{2}$ cents per franc?

132. What must be paid to effect an insurance of $15000, at a premium of $2\frac{5}{8}$ per cent., the charge for the policy being $3.00?

133. A merchant bought 127 bales of cotton, each weighing 375 lb. at $28 per bale, and paid for freight $17.50, for insurance $10.25, for drayage $5.00, and for advertising, and other incidental expenses, $7.50. How much, and what per cent. did he gain by selling the whole at $11\frac{1}{2}$ cents per pound, allowing 36 lb. tare per bale?

134. If a merchant of New York remits money to London at $4.84 per £, thence to Lisbon at 5s. 8d. per milree, thence to Paris at 460 rees for 3 francs, thence to Amsterdam at 2 francs for 37 pfenings Flemish, what amount of Flemish money can be paid with a remittance of $8321?

135. The poll tax being $1.25, and the property tax $\frac{7}{10}$ per cent., what amount of tax must be paid for $8812.50 real estate, $$725\frac{7}{8}$ personal property, and 2 polls?

136. What is the compound interest of $4759 for 3 yr. 11 mo. 18 dy., at 6 per cent.?

137. Bought a house for $7500, paying $\frac{1}{3}$ in cash, $750 in 3 months, $1100 in 12 months, $2250 in 17 months, and $500 in 2 years. How much remained to be paid in 3 years, computing interest at $5\frac{1}{2}$ per cent?

138. What number is that to which if 3 times its square be added, the sum will be 100?

139. Three men purchased a house in company. A. paid $2300, B. $1700, and C. $1950. What yearly income, and what per cent., will each derive from the investment, the house renting for $700, and the taxes and other expenses being $175 per annum?

140. How much sugar at 7 ct. and 8 ct. per lb., must be mixed with 19 lb. at $7\frac{1}{2}$ ct., 24 lb. at $8\frac{3}{4}$ ct., and 11 lb. at 10 ct., to make a mixture of 173 lb., worth 9 cents a pound?

141. Write four 3's, so as to make 1, 14, 34, and 111.

142. If 19 men in $5\frac{1}{2}$ days of $8\frac{1}{2}$ hours build a wall 17 ft. long, 11 ft. high, and 3 ft. thick, in how many days of $9\frac{1}{2}$ hours will 76 men build a wall 100 ft. long 8 ft. high and $3\frac{1}{2}$ ft. thick?

143. If an investment of $1750 amounts to $3325 in 5 yr. 3 mo. what is the annual gain per cent.?

144. A note of $4708, amounted at its settlement, to $5477.758. How long had it been at interest, the rate being 6 per cent.?

145. What principal will yield an annual income of $1650, at 8 per cent.?

146. Bought 117 cwt. 3 qr. 11 lb. of sugar, at $6\frac{1}{4}$ ct. per lb., payable in 4 mos., paying for freight $4.50, cartage, $1.25, and other expenses $5.75. How much, and what per cent. shall I gain, by selling the whole immediately for $900, payable in 90 days, discounting at 6 per cent.?

147. A. hired a house for a year, at the end of 4 months receiving B., at the end of 6 months C., and at the end of 8 months, D., as joint tenants. How should the rent, which was $450, be divided among them?

148. A jeweller mixed 11 lb. of silver, 10 oz. fine, 13 lb. of 11 oz. fine, 7 lb. of 9 oz. fine, 12 lb. pure silver, and 9 lb. alloy. What was the fineness of the mixture?

149. If it cost $7.25 to carry 11 cwt. 3 qr., 44 miles, what will be the freight of 2 T. 17 cwt. 1 qr. for 76 miles?

150. If $13\frac{1}{2}$ tons are carried $262\frac{1}{2}$ miles for $\$153\frac{1}{3}$, how far may $8\frac{5}{8}$ tons be carried for $\$277\frac{1}{2}$?

151. If $17\frac{1}{2}$ yd. of satinet cost $8.75, and $2\frac{3}{4}$ yd. are sold for $1.65, what is the gain per cent.?

152. In a certain partnership A. advanced $\frac{1}{3}$ of the stock, B. $\frac{2}{5}$, and C. the remainder. It is required to divide the gain, which was $7300, properly between them.

153. By what number must 84 be divided, that the quotient may be equal to the product of 84 by $17\frac{3}{5}$?

154. A man had 7 children, whose ages were in arithmetical progression; the eldest was 25, and the youngest was 7. What was the common difference?

155. If 284 men in $5\frac{1}{2}$ days of 11 hours, dig a trench $232\frac{1}{2}$ rods long, $3\frac{2}{3}$ ft. wide, and $2\frac{1}{3}$ ft. deep, in how many days of 9 hours, will 24 men dig a trench $337\frac{1}{2}$ rods long, $5\frac{3}{5}$ ft. wide, and $3\frac{1}{2}$ ft. deep,—the ground, in the latter instance, being but $\frac{4}{7}$ as hard as in the former?

156. A man bought 100 apples at 2 for a cent, and 100 at 3 for 2 cents. What was his gain or loss, by selling the whole at 5 for 2 cents?

157. A merchant of New York purchased in Liverpool, 29 pieces of merino, each $28\frac{1}{2}$ yd., at 2s. 6d., per yd., 38 pieces Scotch gingham, each $31\frac{3}{4}$ yd., at 1s. 9d. per yd., and 17 pieces broadcloth, each 27 yd., at 17s. 5d. per yd. He paid for freight £11 15s., for insurance $9\frac{1}{2}$ per cent., for duties 20 per cent., and for drayage 75 cents. What did the whole cost him in American money, exchange on England being at 10 per cent. premium?

158. A grocer bought 7 cwt. 1 qr. 5 lb. of sugar, for $58.25, and lost $8\frac{1}{2}$ per cent. by its sale. At what price did he sell it per pound?

159. If 60 lb. of bread serve 9 persons 5 days, how much will it cost per year, to supply 843 persons, when 9 lb. of bread can be bought for 25 cents?

160. The notes of the Merchants' Bank being at a discount of $7\frac{1}{2}$ per cent., how much Merchants' Bank money must be employed to pay a debt of $763?

161. How much tea at $62\frac{1}{2}$ ct. per lb., and chocolate, at $12\frac{1}{2}$ ct per lb., of each an equal quantity, must be given in exchange for $128\frac{3}{4}$ gal. of vinegar, at 16 ct. per gal., and $11\frac{5}{8}$ bu. of wheat, at $1.10 per bushel.

162. A. can mow an acre of grass in $7\frac{1}{4}$ hours, and B. in $8\frac{4}{5}$ hours. In what time will they mow it together?

163. Two men, who are 59 miles apart, travel towards each other. A starts 1 hour before B., and travels $10\frac{1}{2}$

miles in 3 hours: B. goes $5\frac{1}{4}$ miles in 2 hours. How far will each travel before they meet?

164. Three men purchase a vessel. A. pays $\frac{4}{13}$, B. $\frac{3}{10}$, and C. the remainder, which was \$2550. What was C.'s share, and how much did A. and B. pay?

165. A stone, dropped from the top of a precipice, reached the ground in $3\frac{1}{2}$ seconds. What was the height?

166. What is the least number, which, being divided by 2, 3, 4, 5, 6, 7, 8, or 9, leaves a remainder 1?

167. The ages of a man and wife at the time of marriage, were in the ratio of 11 to 9; 8 years afterwards they were as 15 to 13. Required the age of each on the wedding day.

168. Find the value of x in the equation,

$$x^5 + 4x^3 - 2x^2 = 4.$$

169. What is the sum of 13 terms of a geometrical progression, whose first term is 2181, and ratio $\frac{1}{3}$?

170. A grazier having two equal flocks of sheep, sells 17 from one, and 95 from the other, when he finds one flock contains three times as many as the other. How many were there originally in each?

171. A footman engaged to serve a year for \$72 and a livery, but leaving at the end of 5 months, he received only \$24 and the livery. What was the price of the suit?

172. At the commencement of business, A. had twice as much money as B., but at the expiration of a year, A. having gained $17\frac{6}{7}$ per cent., and B. having lost \$500, A.'s stock was $2\frac{3}{4}$ times B.'s. Required the original stock of each?

173. A thief travelling $12\frac{1}{2}$ miles in $2\frac{1}{2}$ hours, has been absent $7\frac{3}{4}$ hours, when a constable starts in pursuit, going 11 miles to the thief's 10. In what time will the thief be overtaken?

174. A cistern has 4 pipes, the first of which will fill it in 2 hours, the second in $2\frac{1}{2}$ hours, the third in $2\frac{3}{4}$ hours, and the fourth in $3\frac{1}{3}$ hours. In what time will the cistern be filled, if they are all opened at once?

175. Bought a piece of cloth, which proved to be but $\frac{5}{6}$ as long as it was marked, but I find that if I sell it at \$5.94

per yd, I shall gain 10 per cent. on the prime cost. What did I give per yard?

176. A person left by will to his widow $250 per annum, to his eldest son $6500, to his youngest son $6000, and to his daughter $5500. He had been in business 18 years, commencing with $1500. How much did he save per annum, his earnings having been invested at 5 per cent. compound interest?

177. Bought 29 pieces of broadcloth, each $31\frac{1}{2}$ yd., at $4.75 per yd., and 37 pieces of cassimere, each $34\frac{3}{4}$ yd., at $1.50 per yd., the whole cost to be divided into 3 equal payments at 4, 8, and 12 months. How much should be deducted from the bill upon paying cash, if money is worth 8 per cent. per annum?

178. My expenditure having exceeded my income by 15 per cent., I find that by saving $\frac{1}{8}$ of my income for the succeeding year, I can supply the deficiency with interest, and have $4.60 left. What is my income?

179. Find 11 terms of a Harmonical Progression, two of the terms being 4 and 8.

180. Bought goods amounting to $2500, at 12 months credit, at the end of 3 months $400 were paid, at the end of 6 months $500, and at the end of 8 months $200. When will the balance of $1400 become due?

181. Two persons of the same age conversing together, one said to the other, "My eldest son is just half as old as I am, and if $88\frac{1}{2}$ is added to the continual product of our three ages, the sum will be 52000." What was the age of each?

182. What is the present worth of the reversion of an estate valued at $10000, which comes into possession on the death of an individual now 37 years of age, the average duration of life at that age, being 29 years?

183. What sum of money must a man lay up annually, commencing at 21 years of age, to be worth $30000 when he is 50 years old,—the investments being made at 6 per cent. compound interest?

184. What sum of money at 6 per cent. simple interest,

saved annually, will amount to the same as the preceding example?

185. A., B., and C. meeting on the road, agreed to dine together. A. furnished 5 loaves, B. 3 loaves, and C., having no bread, paid 8 pieces of money for his share. How should the money be divided between A. and B.?

186. A horse is tied to a stake, so that he can graze $\frac{3}{4}$ of an acre. What is the length of the rope?

187. A mechanic hired a certain number of men at $1.00 per day, $\frac{2}{3}$ as many at 75 cents, and $\frac{3}{4}$ as many at 50 cents as at 75 cents. What was the number of each, the daily wages of the whole being $21.00?

188. How many drops of water would fill a demijohn that holds 6 gal. 1 qt. 1 pt.?

189. If 12 oxen eat $3\frac{1}{3}$ acres of grass in 4 weeks, and 21 oxen eat 10 acres of the like pasture in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the grass being at first equal on every acre, and growing uniformly?

This example, which is taken from Newton's Universal Arithmetic, can be solved most readily by making three distinct questions.

(1.) If 12 oxen eat $\frac{10}{3}$ acres of grass, with the growth, in 4 weeks, how many oxen will eat 24 acres, with 4 weeks' growth, in 18 weeks?

$$\begin{array}{r|l} \cancel{3} & \\ \hline 12 & — \\ 4 & \cancel{18} \\ \hline 2\ \ \cancel{24} & \cancel{10}5 \end{array}$$

Stating the question by proportion, we obtain

$$\frac{12 \times 4 \times 2}{5} = 19\tfrac{1}{5} \text{ oxen, for the answer.}$$

(2.) If 21 oxen eat 10 acres with the growth, in 9 weeks, how many oxen will eat 24 acres with 9 weeks' growth, in 18 weeks?

$$\begin{array}{r|l} 21 & — \\ \cancel{9} & \cancel{18} \\ \hline 6\ \ \cancel{24} & \cancel{10}5 \end{array}$$

The answer, found as before, is

$$\frac{21 \times 6}{5} = 25\tfrac{1}{5}.$$

Now, if $19\frac{1}{5}$ oxen in 18 weeks, eat 24 acres with 4 weeks' growth, and $25\frac{1}{5}$ oxen in the same time, eat the same number of acres with 9 weeks' growth, 5 weeks' growth on 24 acres will support 6 oxen 18 weeks. Then,

(3.) If 6 oxen in 18 weeks eat 5 weeks' growth, how many oxen in the same time will eat 9 weeks' growth?

$$\begin{array}{c|c} 6 & \text{—} \\ \hline \text{—} & \text{—} \\ 9 & 5 \end{array}$$

The answer is $10\frac{4}{5}$. We have already found that $25\frac{1}{5}$ oxen in 18 weeks, will eat 24 acres with 9 weeks' growth, and if we add the number which would eat the growth of the remaining 9 weeks, we obtain 36 oxen for the answer sought.

190. A road has been constructed through a farmer's land, taking $1\frac{1}{4}$ acres, worth $75 per acre. In addition to the loss of his land, he is obliged to expend $100 in building a fence, which must be renewed every 12 years. What damages should he receive, money being worth 6 per cent. compound interest?

191. If 15 oxen eat $4\frac{1}{2}$ acres of grass in 2 weeks, and 24 oxen eat $14\frac{2}{5}$ acres in 5 weeks, how many oxen will eat 48 acres in 8 weeks, the grass being at first equal on every acre, and growing uniformly?

192. Find 6 weights with which any number of pounds, from 1 to 364, can be weighed.

193. What is the sum of all the divisors of 33550336?*

194. Four men bought a grindstone 40 inches in diameter, each contributing an equal amount. How much of the diameter ought each to grind away?

195. If the wheel of a carriage turn once and a half in going a rod and a half, how often will it turn in $17\frac{3}{5}$ miles?

196. The square root of $\frac{2}{9}$ of 3 times $\frac{2}{7}$ of 3 times a man's age is 4. What is his age?

* Any number that is equal to the sum of all its aliquot parts, is called a PERFECT NUMBER. The only perfect numbers known, are,

6
28
496
8128
33550336
8589869056
137438691328
2305843008139952128
2417851639228158837784576
9903520314282971830448816128

197. What is the difference between the area of a circle whose circumference is $157\frac{2}{25}$ feet, and the area of the greatest square that can be inscribed in it?

198. A. and B. are on opposite sides of a field that is 120 rods in diameter, and commence travelling around it in the same direction. How many times will each go round the field before the slower is overtaken, A. going 39 rods in 3 minutes, and B. $66\frac{2}{3}$ rods in 5 minutes?

199. What number is that which is divisible by 11, but if divided by any number less than 11, leaves 1 remainder?

200. The Peak of Teneriffe is about $2\frac{1}{2}$ miles high. How far may it be seen at sea from the main-top of a vessel, 60 feet above the surface of the water?

201. A wall $5\frac{1}{2}$ ft. high, and $1\frac{3}{4}$ ft. thick, is to be built around a garden that is 180 ft. long, and 60 ft. wide, leaving space for two gates, each $6\frac{3}{4}$ ft. wide. How many bricks, each 8 in. long, 4 in. wide, and 2 in. thick, will be required?

202. A man sold a horse for $65.25, thereby gaining as much per cent. as the horse cost him. What did he give for the horse?

203. "One evening I chanced with a tinker to sit,
Whose tongue ran a great deal too fast for his wit:
He talked of his art with abundance of mettle,
So I asked him to make me a flat bottomed kettle.
Let the top and the bottom diameters be
In just such proportion as five is to three:
Twelve inches the depth I proposed, and no more,
And to hold in ale gallons, seven less than a score.
He promised to do it, and straight to work went,
But when he had done it he found it too scant.
He altered it then, but too big he had made it,
And when it held right, the diameters failed it:
Thus making it often too big and too little,
The tinker at last had quite spoiled his kettle,
But declared he would bring his said promise to pass,
Or else that he'd spoil every ounce of his brass.
Now to keep him from ruin, I pray find him out
The diameter's length, for he'll ne'er do't without."

204. Two vessels are 30 miles apart, and are sailing, the

first with, and the second against a current of $2\frac{1}{2}$ miles per hour. In still water, each would sail 7 miles per hour. In what time will they meet, and what will be the distance of each from its present position?

205. At 6 per cent. compound interest, what annual payment will extinguish a debt of $3500000 in 50 years?

206. The Winchester bushel is $18\frac{1}{2}$ inches wide and 8 inches deep. What are the dimensions of a tub of similar form that holds $6\frac{1}{4}$ bushels?

207. How much should an annual rent be increased, to amount in 17 years, to the same as $2500 at 6 per cent. compound interest?

208. At what time between half past 7 and 8 o'clock, are the hour and minute hands exactly 13 minutes apart?

209. An estate of $4896 is to be divided in such manner that the widow shall receive $\frac{3}{5}$ as much as the son, and the son $\frac{7}{8}$ of $2\frac{1}{2}$ times as much as the daughter. What must each receive?

210. Three men bought a grindstone 50 inches in diameter, for which A. paid 75 cents, B. $1.50, and C. $2.00. What part of the diameter ought each to wear away, allowing the diameter of the axle to be 2 inches?

211. A vessel of 400 tons has a keel 48 feet long. What length of keel has a vessel of 750 tons, that is built on the same model?

212. The commissioners of a certain county are about building a new court house, which will cost $75000. They hire money for the purpose at an annual interest of 5 per cent. and propose to pay the debt thus incurred by 50 equal annual instalments. What amount must be paid each year?

213. Sold a quantity of sugar for $150, thereby losing 15 per cent., but I ought to have gained as much per cent. as the sugar cost. What was my total loss?

214. An estate of $20000 is to be divided between two sons in the following manner: the elder is to receive $100 the first month, $300 the second month, &c., in arithmetical progression—and the younger is to receive $1000 per month, until the whole is paid. What is the share of each, and how long will they be in receiving it?

215. What is the quotient of $90\frac{3}{5}$ times half a dozen dozen, divided by $121\frac{1}{7}$ times six dozen dozen?

216. A ladder standing upright against a wall reaches the top, but the foot being removed 12 feet from the wall, it reaches to a point 6 feet from the top. Required the length of the ladder, and the height of the wall.

217. There are two cannon balls, one weighing 28 pounds, and the other 9 pounds. What is the diameter of the greater—that of the less being 5 inches?

218. If one-third of six were three,
What would the fourth of twenty be?

219. A man has a perpetual income of $350 per annum, which he desires to exchange for a life annuity. Supposing money to be worth 6 per cent. a year, what annuity must he receive, it being probable that he will live 16 years?

220. A man hires a farm for 15 years, and expends $1200 on improvements which yield him 7 per cent. per annum. What is the present worth of the gain or loss on his expenditure?

221. Find the value of x from the equation $\sqrt[3]{x} = \sqrt[2]{5}$.

222. A merchant received on consignment, three bales of sheeting, marked A., B., and C. A. contained 420 yards of a quality 15 per cent. better than B., B. contained 380 yards of a quality 10 per cent. poorer than C., and C. contained 450 yards. The whole were sold together at $12\frac{1}{2}$ cents per yard; how much should be credited to each, after deducting $2\frac{1}{2}$ per cent. commission?

223. An estate was offered for sale for $12000, but the price appearing too high, the tenant took a lease for 25 years at $800 per annum. How much did he gain or lose, estimating compound interest at 6 per cent.?

224. Reduce $\sqrt{4} \times \sqrt{7}$ to a continued fraction.

225. A man wishes to give his son, who is now 5 years old, $20000 when he is 21 years old. How much must he invest annually for that purpose, at 6 per cent. compound interest?

226. A farmer has a stack of hay, from which he sells a quantity which is to the quantity remaining, as 4 to 5. He

then uses 15 loads and finds that the quantity left, is to the quantity sold, as 1 to 2. Required the number of loads at first in the stack.

227. There are two numbers, the greater of which is 3 times the less, and the sum of their squares is $8\frac{1}{3}$ times the sum of the numbers. What are they?

228. Find two numbers in the proportion of 4 to 7, the square of whose product is 63504.

229. A farm of 750 acres is divided among three persons. B. has as much as A. and C. wanting 10 acres, and the shares of A. and C. are to each other in the proportion of 7 to 3. Required the share of each.

230. If 11 oxen eat $24\frac{4}{9}$ acres of grass in 5 weeks, and 10 oxen eat $19\frac{1}{21}$ acres in 4 weeks, how many acres of similar pasture will 42 oxen eat in 7 weeks, the grass growing uniformly?

231. What number is that which is 169 greater than the greatest square number below, and 114 less than the least square number above itself?

232. A. and B. can do $\frac{1}{10}$ of a piece of work in a day, B. and C. can do $\frac{1}{5}$ of it in $2\frac{1}{4}$ days, and A. and C. can do $\frac{1}{3}$ of it in $4\frac{1}{11}$ days. In what time would each do it alone, and in what time would it be done if they all worked together?

233. Find the least 3 integers, such that $\frac{3}{8}$ of the first, $\frac{5}{14}$ of the second, and $\frac{7}{20}$ of the third, shall be equal.

234. An estate of $20000 is to be divided among three sons, two daughters, and a widow; each son is to receive 3 times as much as a daughter, and each daughter twice as much as the widow. Required the share of each.

235. An usurer left his fortune to be disposed of in the following manner: To A. $\frac{2}{5}$, to B. $\frac{3}{10}$, to C. $\frac{1}{8}$, to D. $\frac{1}{20}$, to E. $\frac{1}{40}$, to F. $\frac{1}{50}$, and the remainder, which was $800, was to be paid to C. What was the whole estate, and what was the share of each?

236. For the lease of a certain estate, A offers $150 premium, and $300 rent per annum; B. offers $400 premium, and $250 per annum; C. offers $650 premium, and $200

per annum; and D. offers $1800 in ready money. Whose offer is the best, and what is the difference between them, computing 5 per cent. compound interest?

237. Which is of the greater value, the income of an estate of $500 a year for 15 years to come, or the reversion of the same estate for ever, at the expiration of the 15 years, interest at 6 per cent.?

238. If a ball were put in motion by a force which would drive it 12 miles the first hour, 10 miles the second, and so on in geometrical progression, what distance would it go in the whole?

239. What is the least number, which, if divided by 2, will leave 1 remainder, by 3 will leave 2, by 4 will leave 3, by 5 will leave 4, by 6 will leave 5, but by 7, will leave no remainder?

240. Required the least three numbers, which, divided by 20, will leave 19 remainder, if divided by 19 will leave 18, and so on, (always leaving one less than the divisor,) to unity.

241. A trader offers to receive a young man as partner, proposing, if he will advance $500, to allow him $200 per annum; if he will advance $1000, to allow $275 per annum; and if he advance $1500, he will allow $350 per annum. What per cent. is offered for the use of the money, and how much for the young man's time?

242. Arnold's stock in the firm of Arnold and Benton, exceeded Benton's by $2000; Arnold's money continued in trade 5 months, and Benton's 9 months. Required each man's investment, the gain being equally divided.

243. A shepherd sold to one man half his flock and half a sheep, to a second half the remainder and half a sheep, and to a third half the remainder and half a sheep, when he had 20 left. How many had he at first?

END OF PART SECOND.

USE OF THE TABLES.

I. PRIME AND COMPOSITE NUMBERS.

The hundreds are placed at the head of the table, and the tens and units at the left hand. Then, if it be desired to find 5609, look for 56 at the top, and 09 at the side, and at the angle of meeting we find 71, which is one of the factors. Dividing by 71 we obtain 79, which being a prime number is the only remaining factor.

For numbers below 1000, all the factors are given. For all *odd* numbers above 1000, one or more factors will be found in the table, which will reduce the number to a prime or to some composite number less than 1000, when the remaining factors can be found.

Primes are indicated by a dash (—). The index written against any number, indicates the number of times it is employed as a factor. Thus, $3^2 = 3 \times 3$; $5^3 = 5 \times 5 \times 5$.

II. COMPOUND INTEREST.

The amount of any principal, is found by multiplying the principal by the amount of \$1.00 for the time. If the time consists of years, months, and days, the amount for the given number of years, must be multiplied by the amount of \$1.00 for the remaining months and days.

The present worth at compound interest, of any sum due at a future time, is found by dividing the given sum by the amount of \$1.00 for the time.

III. THE AMOUNT OF ANNUITIES.

To find the amount of any annuity; multiply the given annuity by the amount of an annuity of \$1.00 for the time. If the annuity is payable semi-annually, take the amount for twice the time, at half the rate.

IV. PRESENT WORTH OF ANNUITIES.

Multiply the given annuity by the present worth of an annuity of \$1.00 for the given time.

The present worth being given, to find the annuity; divide the present worth by the present worth of an annuity of \$1.00 for the same time.

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	0	1	2	3	4	5	6	7	8	9
00	—	$2^2.5^2$	$2^3.5^2$	$2^2.3.5^2$	$2^4.5^2$	$2^2.5^3$	$2^3.3.5^2$	$2^2.5^2.7$	$2^5.5^2$	$2^2.3^2.5^2$
01	—	—	3.67	7.43	—	3.167	—	—	$3^2.89$	17.53
02	—	2.3.17	2.101	2.151	2.3.67	2.251	2.7.43	$2.3^3.13$	2.401	2.11.41
03	—	—	7.29	3.101	13.31	—	$3^2.67$	19.37	11.73	3.7.43
04	2^2	$2^3.13$	$2^2.3.17$	$2^4.19$	$2^2.101$	$2^3.7.9$	$2^2.151$	$2^6.11$	$2^2.3.67$	$2^3.113$
05	—	3.5.7	5.41	5.61	$3^4.5$	5.101	5.11^2	3.5.47	5.7.23	5.181
06	2.3	2.53	2.103	$2.3^2.17$	2.7.29	2.11.23	2.3.101	2.353	2.13.31	2.3.151
07	—	—	$3^2.23$	—	11.37	3.13^2	—	7.101	3.269	—
08	2^3	$2^2.3^3$	$2^4.13$	$2^2.7.11$	$2^3.3.17$	$2^2.127$	$2^5.19$	$2^2.3.59$	$2^3.101$	$2^2.227$
09	3^2	—	11.19	3.103	—	—	3.7.29	—	—	$3^2.101$
10	2.5	2.5.11	2.3.5.7	2.5.31	2.5.41	2.3.5.17	2.5.61	2.5.71	$2.3^4.5$	2.5.7.13
11	—	3.37	—	—	3.137	7.73	13.47	$3^2.79$	—	—
12	$2^2.3$	$2^4.7$	$2^2.53$	$2^3.3.13$	$2^2.103$	2^9	$2^2.3^2.17$	$2^3.89$	$2^2.7.29$	$2^4.3.19$
13	—	—	3.71	—	7.59	$3^3.19$	—	23.31	3.271	11.83
14	2.7	2.3.19	2.107	2:157	$2.3^2.23$	2.257	2.307	2.3.7.17	2.11.37	2.457
15	3.5	5.23	5.43	$3^2.5.7$	5.83	5.103	3.5.41	5.11.13	5.163	3.5.61
16	2^4	$2^2.29$	$2^3.3^3$	$2^2.79$	$2^5.13$	$2^2.3.43$	$2^3.7.11$	$2^2.179$	$2^4.3.17$	$2^2.229$
17	—	$3^2.13$	7.31	—	3.139	11.47	—	3.239	19.43	7.131
18	2.3^2	2.59	2.109	2.3.53	2.11.19	2.7.37	2.3.103	2.359	2.409	$2.3^3.17$
19	—	7.17	3.73	11.29	—	3.173	—	—	$3^2.7.13$	—
20	$2^2.5$	$2^3.3.5$	$2^2.5.11$	$2^6.5$	$2^2.3.5.7$	$2^3.5.13$	$2^2.5.31$	$2^4.3^2.5$	$2^2.5.41$	$2^3.5.23$
21	3.7	11^2	13.17	3,107	—	—	$3^3.23$	7.103	—	3.307
22	2.11	2.61	2.3.37	2.7.23	2.211	$2.3^2.29$	2.311	2.19^2	2.3.137	2.461
23	—	3.41	—	17.19	$3^2.47$	—	7.89	3.241	—	13.71
24	$2^3.3$	$2^2.31$	$2^5.7$	$2^2.3^4$	$2^3.53$	$2^2.131$	$2^4.3.13$	$2^2.181$	$2^3.103$	$2^2.3.7.11$
25	5^2	5^3	$3^2.5^2$	$5^2.13$	$5^2.17$	$3.5^2.7$	5^4	$5^2.29$	$3.5^2.11$	$5^2.37$
26	2.13	$2.3^2.7$	2.113	2.163	2.3.71	2.263	2.313	$2.3.11^2$	2.7.59	2.463
27	3^3	—	—	3.109	7.61	17.31	3.11.19	—	—	$3^2.103$
28	$2^2.7$	2^7	$2^2.3.19$	$2^3.41$	$2^2.107$	$2^4.3.11$	$2^2.157$	$2^3.7.13$	$2^2.3^2.23$	$2^5.29$
29	—	3.43	—	7.47	3.11.13	23^2	17.37	3^6	—	—
30	2.3.5	2.5.13	2.5.23	2.3.5.11	2.5.43	2.5.53	$2.3^2.5.7$	2.5.73	2.5.83	2.3.5.31
31	—	—	3.7.11	—	—	$3^2.59$	—	17.43	3.277	$7^2.19$
32	2^5	$2^2.3.11$	$2^3.29$	$2^2.83$	$2^4.3^3$	$2^2.7.19$	$2^3.79$	$2^2.3.61$	$2^6.13$	$2^2.233$
33	3.11	7.19	—	$3^2.37$	—	13.41	3.211	—	$7^2.17$	3.311
34	2.17	2.67	$2.3^2.13$	2.167	2.7.31	2.3.89	2.317	2.367	2.3.139	2.467
35	5.7	$3^3.5$	5.47	5.67	3.5.29	5.107	5.127	$3.5.7^2$	5.167	5.11.17
36	$2^2.3^2$	$2^3.17$	$2^2.59$	$2^4.3.7$	$2^2.109$	$2^3.67$	$2^2.3.53$	$2^5.23$	$2^2.11.19$	$2^2.3^2.13$
37	—	—	3.79	—	19.23	3.179	$7^2.13$	11.67	$3^3.31$	—
38	2.19	2.3.23	2.7.17	2.13^2	2.3.73	2.269	2.11.29	$2.3^2.41$	2.419	2.7.67
39	3.13	—	—	3.113	—	$7^2.11$	$3^2.71$	—	—	3.313
40	$2^3.5$	$2^2.5.7$	$2^4.3.5$	$2^2.5.17$	$2^3.5.11$	$2^2.3^3.5$	$2^7.5$	$2^2.5.37$	$2^3.3.5.7$	$2^2.5.47$
41	—	3.47	—	11.31	$3^2.7^2$	—	—	3.13.19	29^2	—
42	2.3.7	2.71	2.11^2	$2.3^2.19$	2.13.17	2.271	2.3.107	2.7.53	2.421	2.3.157
43	—	11.13	3^5	7^3	—	3.181	—	—	3.281	23.41
44	$2^2.11$	$2^4.3^2$	$2^2.61$	$2^3.43$	$2^2.3.37$	$2^5.17$	$2^2.7.23$	$2^3.3.31$	$2^2.211$	$2^4.59$
45	$3^2.5$	5.29	5.7^2	3.5.23	5.89	5.109	3.5.43	5.149	5.13^2	$3^3.5.7$
46	2.23	2.73	2.3.41	2.173	2.223	2.3.7.13	2.17.19	2.373	$2.3^2.47$	2.11.43
47	—	3.7^2	13.19	—	3.149	—	—	$3^2.83$	7.11^2	—
48	$2^4.3$	$2^2.37$	$2^3.31$	$2^2.3.29$	$2^6.7$	$2^2.137$	$2^3.3^4$	$2^2.11.17$	$2^4.53$	$2^2.3.79$
49	7^2	—	3.83	—	—	$3^2.61$	11.59	7.107	3.283	13.73

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	0	1	2	3	4	5	6	7	8	9
50	2.5^2	$2.3.5^2$	2.5^3	$2.5^2.7$	$2.3^2.5^2$	$2.5^2.11$	$2.5^2.13$	$2.3.5^3$	$2.5^2.17$	$2.5^2.19$
51	3.17	——	——	$3^3.13$	11.41	19.29	3.7.31	——	23.37	3.317
52	$2^2.13$	$2^3.19$	$2^2.7.9$	$2^5.11$	$2^2.113$	$2^3.3.23$	$2^2.163$	$2^4.47$	$2^2.3.71$	$2^3.7.17$
53	——	$3^2.17$	11.23	——	3.151	7.79	——	3.251	——	——
54	2.3^3	2.7.11	2.127	2.3.59	2.227	2.277	2.3.109	2.13.29	2.7.61	$2.3^2.53$
55	5.11	5.31	3.5.17	5.71	5.7.13	3.5.37	5.131	5.151	$3^2.5.19$	5.191
56	$2^3.7$	$2^2.3.13$	2^8	$2^2.89$	$2^3.3.19$	$2^2.139$	$2^4.41$	$2^2.3^3.7$	$2^3.107$	$2^2.239$
57	3.19	——	——	3.7.17	——	——	$3^2.73$	——	——	3.11.29
58	2.29	2.79	2.3.43	2.179	2.229	$2.3^2.31$	2.7.47	2.379	2.3.11.13	2.479
59	——	3.53	7.37	——	$3^3.17$	13.43	——	3.11.23	——	7.137
60	$2^2.3.5$	$2^5.5$	$2^2.5.13$	$2^3.3^2.5$	$2^2.5.23$	$2^4.5.7$	$2^2.3.5.11$	$2^3.5.19$	$2^2.5.43$	$2^6.3.5$
61	——	7.23	$3^2.29$	19^2	——	3.11.17	——	——	3.7.41	31^2
62	2.31	2.3^4	2.131	2.181	2.3.7.11	2.281	2.331	2.3.127	2.431	2.13.37
63	$3^2.7$	——	——	3.11^2	——	——	3.13.17	7.109	——	$3^2.107$
64	2^6	$2^2.41$	$2^3.3.11$	$2^2.7.13$	$2^4.29$	$2^2.3.47$	$2^3.83$	$2^2.191$	$2^5.3^3$	$2^2.241$
65	5.13	3.5.11	5.53	5.73	3.5.31	5.113	5.7.19	$3^2.5.17$	5.173	5.193
66	2.3.11	2.83	2.7.19	2.3.61	2.233	2.283	$2.3^2.37$	2.383	2.433	2.3.7.23
67	——	——	3.89	——	——	$3^4.7$	23.29	13.59	3.17^2	——
68	$2^2.17$	$2^3.3.7$	$2^2.67$	$2^4.23$	$2^2.3^2.13$	$2^3.71$	$2^2.167$	$2^8.3$	$2^2.7.31$	$2^3.11^2$
69	3.23	13^2	——	$3^2.41$	7.67	——	3.223	——	11.79	3.17.19
70	2.5.7	2.5.17	$2.3^3.5$	2.5.37	2.5.47	2.3.5.19	2.5.67	2.5.7.11	2.3.5.29	2.5.97
71	——	$3^2.19$	——	7.53	3.157	——	11.61	3.257	13.67	——
72	$2^3.3^2$	$2^2.43$	$2^4.17$	$2^2.3.31$	$2^3.59$	$2^2.11.13$	$2^5.3.7$	$2^2.193$	$2^3.109$	$2^2.3^5$
73	——	——	3.7.13	——	11.43	3.191	——	——	$3^2.97$	7.139
74	2.37	2.3.29	2.137	2.11.17	2.3.79	2.7.41	2.337	$2.3^2.43$	2.19.23	2.487
75	3.5^2	$5^2.7$	$5^2.11$	3.5^3	$5^2.19$	$5^2.23$	$3^3.5^2$	$5^2.31$	$5^3.7$	$3.5^2.13$
76	$2^2.19$	$2^4.11$	$2^2.3.23$	$2^3.47$	$2^2.7.17$	$2^6.3^2$	$2^2.13^2$	$2^3.97$	$2^2.3.73$	$2^4.61$
77	7.11	3.59	——	13.29	$3^2.53$	——	——	3.7.37	——	——
78	2.3.13	2.89	2.139	$2.3^3.7$	2.239	2.17^2	2.3.113	2.389	2.439	2.3.163
79	——	——	$3^2.31$	——	——	3.193	7.97	19.41	3.293	11.89
80	$2^4.5$	$2^2.3^2.5$	$2^3.5.7$	$2^2.5.19$	$2^5.3.5$	$2^2.5.29$	$2^3.5.17$	$2^2.3.5.13$	$2^4.5.11$	$2^2.5.7^2$
81	3^4	——	——	3.127	13.37	7.83	3.227	11.71	——	$3^2.109$
82	2.41	2.7.13	2.3.47	2.191	2.241	2.3.97	2.11.31	2.17.23	$2.3^2.7^2$	2.491
83	——	3.61	——	——	3.7.23	11.53	——	$3^3.29$	——	——
84	$2^2.3.7$	$2^3.23$	$2^2.71$	$2^7.3$	$2^2.11^2$	$2^3.73$	$2^2.3^2.19$	$2^4.7^2$	$2^2.13.17$	$2^3.3.41$
85	5.17	5.37	3.5.19	5.7.11	5.97	$3^2.5.13$	5.137	5.157	3.5.59	5.197
86	2.43	2.3.31	2.11.13	2.193	2.3^5	2.293	2.7^3	2.3.131	2.443	2.17.29
87	3.29	11.17	7.41	$3^2.43$	——	——	3.229	——	——	3.7.47
88	$2^3.11$	$2^2.47$	$2^5.3^2$	$2^2.97$	$2^3.61$	$2^2.3.7^2$	$2^4.43$	$2^2.197$	$2^3.3.37$	$2^2.13.19$
89	——	$3^3.7$	17^2	——	3.163	19.31	13.53	3.263	7.127	23.43
90	$2.3^2.5$	2.5.19	2.5.29	2.3.5.13	$2.5.7^2$	2.5.59	2.3.5.23	2.5.79	2.5.89	$2.3^2.5.11$
91	7.13	——	3.97	17.23	——	3.197	——	7.113	$3^4.11$	——
92	$2^2.23$	$2^6.3$	$2^2.73$	$2^3.7^2$	$2^2.3.41$	$2^4.37$	$2^2.173$	$2^3.3^2.11$	$2^2.223$	$2^5.31$
93	3.31	——	——	3.131	17.29	——	$3^2.7.11$	13.61	19.47	3.331
94	2.47	2.97	$2.3.7^2$	2.197	2.13.19	$2.3^3.11$	2.347	2.397	2.3.149	2.7.71
95	5.19	3.5.13	5.59	5.79	$3^2.5.11$	5.7.17	5.139	3.5.53	5.179	5.199
96	$2^5.3$	$2^2.7^2$	$2^3.37$	$2^2.3^2.11$	$2^4.31$	$2^2.149$	$2^3.3.29$	$2^2.199$	$2^7.7$	$2^2.3.83$
97	——	——	$3^3.11$	——	7.71	3.199	17.41	——	3.13.23	——
98	2.7^2	$2.3^2.11$	2.149	2.199	2.3.83	2.13.23	2.349	2.3.7.19	2.449	2.499
99	$3^2.11$	——	13.23	3.7.19	——	——	3.233	17.47	29.31	$3^3.37$

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
01	7	3	–	–	3	19	–	3	–	–	3	11	31	3	7^4	41	3	37	–	3
03	17	–	3	–	23	3	7	13	3	11	–	3	–	7	3	–	19	3	–	–
05	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
07	19	3	17	–	3	11	–	3	13	–	3	7	–	3	29	23	3	–	7	3
09	–	–	3	7	–	3	–	–	3	23	7	3	47^2	–	3	13	–	3	53^2	–
11	3	11	7	3	17	–	3	29	–	3	–	–	3	–	–	3	7	–	3	41
13	–	3	–	13	3	17	–	3	7	–	3	–	–	3	19	7	3	–	29	3
15	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5
17	3	–	–	3	13	37	3	17	23	3	–	29	3	7	–	3	–	11	3	–
19	–	3	23	–	3	7	–	3	17	19	3	13	7	3	41	11	3	–	–	3
21	–	19	3	–	7	3	–	–	3	17	43	3	–	11	3	–	–	3	7	23
23	3	–	–	3	–	–	3	–	–	3	7	11	3	23	–	3	43	7	3	37
25	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3
27	13	7	3	–	–	3	–	11	3	41	–	3	17	13	3	7	37	3	11	–
29	3	–	–	3	–	11	3	7	31	3	–	–	3	17	7	3	11	–	3	29
31	–	3	–	11^3	3	–	7	3	–	–	3	–	23	3	11	–	3	–	19	3
33	–	11	3	31	–	3	23	–	8	–	19	3	7	–	3	17	–	3	–	7
35	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
37	17	3	–	7	3	29	–	3	11	13	3	–	–	3	–	43	3	7	–	3
39	–	17	3	13	–	3	11	37	3	7	–	3	–	–	3	–	7	3	17	–
41	3	7	17	3	11	23	3	–	7	3	13	–	3	–	–	3	19	–	3	17
43	7	3	11	17	3	–	31	3	19	29	3	–	–	3	7	–	3	13	–	3
45	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5
47	3	31	29	3	–	7	3	–	–	3	23	19	3	–	–	3	–	41	3	7
49	–	3	–	19	3	–	17	3	43^2	–	3	7	13	3	31	–	3	–	7	3
51	–	–	3	7	–	3	13	17	3	–	7	3	–	–	3	–	11	3	–	13
53	3	–	7	3	–	–	3	–	17	3	–	–	3	13	11	3	7	–	3	–
55	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3
57	7	13	3	23	31	3	–	7	3	19	11	3	37	–	3	–	–	3	–	–
59	3	19	–	3	–	–	3	–	11	3	29	17	3	7	–	3	–	31	3	11
61	–	3	13	–	3	7	11	3	–	37	3	–	7	3	23	13	3	11	–	3
63	–	–	3	29	7	3	–	41	3	13	–	3	31	17	3	11	–	3	7	–
65	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
67	11	3	7	–	3	–	–	3	–	7	3	11	–	3	–	17	3	–	47	3
69	–	7	3	37^2	13	3	–	29	3	11	–	3	–	23	3	7	17	3	19	–
71	3	–	31	3	–	–	3	7	–	3	19	13	3	–	7	3	–	17	3	–
73	29	3	19	–	3	11	7	3	–	–	3	41	–	3	–	31	3	47	13	3
75	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5
77	3	11	–	3	7	19	3	–	–	3	31	7	3	–	–	3	–		3	13
79	13	3	–	7	3	–	23	3	–	–	3	–	43	3	37	–	3	7	–	3
81	23	–	3	–	–	3	41^2	13	3	7	–	3	–	–	3	29	7	3	43	11
83	3	7	–	3	–	–	3	–	7	3	–	37	3	–	13	3	–	11	3	19
85	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3
87	–	–	3	19	–	3	7	–	3	–	–	3^7	–	7	3	13	–	3	–	29
89	3	29	–	3	–	7	3	–	–	3	–	11	3	–	19	3	–	–	3	7
91	–	3	–	13	3	37	19	3	31	11	3	7	29	3	47	–	3	–	7	3
93	–	–	3	7	–	3	–	11	3	–	7	3	–	–	3	–	–	3	11	41
95	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5	5	3	5
97	–	3	–	11	3	–	–	3	7	–	3	13^3	–	3	11	7	3	–	–	3
99	7	11	3	–	–	3	–	7	3	–	–	3	11	–	3	23	–	3	13	–

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
01	–	7	11	–	19	3^2	13	–	7	47	–	3	–	11	3^2	7	43	3	–	13
03	7	29	–	3^2	41	31	3	7	–	3	–	11	3^2	13	7	19	–	–	3	–
05	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
07	31	13	3	–	–	7	–	11	3^2	–	–	3	7	59	13	–	17	3^2	11	7
09	17	–	–	3	7	11	3^2	–	13	3	19	7	23	31	–	3^2	11	17	7	–
11	–	17	13	7	3^2	–	23	3	37	–	7	–	–	3^2	11	13	29	7	17	3
13	23	11	7	–	–	3	–	47	31	7	–	3^2	11	19	3	–	7	3	–	17^2
15	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
17	7	3	–	31	17	–	–	7	11	–	13	23	–	3	7	–	3^2	53	–	11
19	–	–	29	–	13	3^2	7	–	19	–	–	3	–	7	3^2	–	31	11	61	–
21	19	–	–	3^2	11	7	17	61^2	–	3	–	13	7	29	–	11	–	–	3	7
23	–	3^2	11	–	7	13	–	17	–	–	3^2	7	41	11	–	–	23	–	7	3^2
25	5	5^5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
27	3	53	7	3	23	–	13	–	43	7	–	–	3	–	19	3^2	7	29	3	13
29	13	7	–	–	3^2	–	19	11	7	–	17	–	–	3^2	43	7	3	–	11	31
31	7	31	3^2	–	47	11	–	7	3	–	29	17	–	61	7	23	11	19	–	–
33	3^2	13	53	11	–	–	7	–	–	19	37	–	17	7	11	3	41	–	3^2	–
35	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
37	–	–	13	47	7	3^2	–	37	3	31	11	7	19	–	17	13	–	3	7	–
39	3	43	41	7	19	–	3	–	11	13	7	–	3^2	–	23	17	–	7	3	11
41	–	3^2	7	13	31	–	11	29	23	7	3^2	41	–	3	–	19	7	11	47	3^2
43	17	7	23	–	11	3	–	19	7	–	13	3	–	43	3	7	–	3^2	29	–
45	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
47	11	3	17	–	3^2	–	7	3	–	–	19	11	31	7	–	–	3	47	37	17
49	–	47	19	17	–	7	41	23	3	11	–	3^2	7	–	3	–	–	3	13	7
51	3^2	23	–	3	7	53	3	11	–	3^2	–	7	13	19	–	31	–	–	7	–
53	43	3	–	7	3	11	13	3^2	–	59	7	–	–	3	61	29	11	7	23	13
55	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
57	3	7	–	3^2	–	–	23	13	7	3	–	–	11	–	–	7	–	67	3	–
59	7	13	–	–	3	–	–	7	17	37	11	–	–	3	7	47	3	–	43	3^2
61	–	29	3	–	–	3	7	–	11	17	31	19	–	7	3	–	59	3^2	–	11
63	3	–	13	19	–	7	11	53	–	3	17	23	7	–	–	3^2	–	11	3	7
65	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
67	–	–	11	7	–	29	19	–	3	–	7	3^2	17	11	3	–	13	7	31	–
69	11	–	7	3	–	43	3	–	53	7	13	11	3	17	41	3	7	19	3^2	–
71	37	7	–	–	13	–	–	3^2	7	11	23	43	–	31	17	7	3^2	13	–	3
73	7	19	3	–	23	3^2	–	7	3	29	–	13	–	–	7	17	–	37	11	–
75	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
77	17	3^2	29	11	19	7	–	3	–	41	3^2	–	7	3	11	23	3	17	–	7
79	–	11	3	31	7	3	13	–	3^2	23	–	7	11	29	3	19	–	3^2	7	13
81	13	–	17	7	59^2	–	3^2	19	–	3	7	37	3	13	–	3^2	31	7	3	17
83	–	3	7	17	3^2	–	29	13	11	7	3	47	–	3^2	–	–	7	–	19	11
85	5	5	5^2	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
87	7	–	19	3	11	17	3	7	13	3^2	61	53	3	41	7	11	43	–	3^2	–
89	–	3	11	–	3	37	7	3^2	–	–	29	59	–	7	67^2	13	3^2	–	–	3
91	11	–	3	–	–	7	–	17	3	13	–	11	7	–	3^2	–	–	3	67	7
93	3	31	37	13	7	–	3	–	17	11	–	7	3^2	23	–	3	13	–	7	–
95	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
97	19	23	7	43	13	11	–	–	3^2	7	17	3	–	–	3	–	7	3^2	59	19
99	3	7	–	11	–	59	3^2	29	7	31	–	13	3	53	11	7	37	–	23	–

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69
01	3	-	7	3^2	11	-	3	-	-	7	17	-	3^2	-	37	11	7	-	3	67
03	-	3^2	11	-	3	-	13	3	7	-	3^2	17	-	11	19	7	31	-	-	3^2
05	7	5	3.5	5	23	3.5	19	7	3^2	5	5	3.5	17	13	3.5	5	5	3^2	5	5
07	3	-	41	29	-	-	3^2	13	-	11	-	31	3	7	43	3^2	-	19	3	-
09	-	13	-	-	3^2	7	71	11	37	19	3	41	7	3^2	13	23	3	-	11	7
11	-	19	3^2	47	7	11	31	-	13	23	-	3^2	-	-	3	17	11	3	7	-
13	3^2	-	13	7	-	37	3	29	-	3^2	7	-	19	59	11	13	17	7	3^2	31
15	17	11	7	5	3.5	5	5	3^2	5	7	3.5	5	11	3.5	5	5	3^2	17	29	3.5
17	29	7	37	13	-	3^2	41	-	7	61	11	3	-	-	3^2	7	13	3	17	-
19	7	-	17	3^2	-	-	3	7	11	3	13	29	3^2	71	7	41	-	-	3	11
21	-	3^2	23	17	13	-	7	3	-	31	3^2	-	-	7	-	-	3	11	19	3^2
23	-	47	3	-	11	7	-	59	3^2	-	19	13	7	-	3	11	37	3^2	-	7
25	5^2	5^2	5^2	5^2	5^2	5^2	3^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	11	3	-	7	3^2	-	17	23	-	-	7	11	13	3^2	-	61	3	7	-	3
29	47	23	3^2	73^2	61	19	13	17	29	7	-	3^2	-	-	3	-	7	3	-	13
31	3^2	7	-	3	-	-	3	11	7	3^2	37	-	31	13	59	7	19	53	3^2	29
33	7	29	-	-	3	11	43	3^2	19	17	3	-	23	3	7	47	3^2	-	-	3
35	19	13	3.5	11	5	3^2	7	31	3.5	5	17	3.5	29	7	3^2	5	5	3.5	5	19
37	23	11	-	3^2	-	7	3	-	13	3	-	17	3^2	-	41	3	-	-	43	7
39	-	3^2	13	19	7	29	-	3	-	-	3^2	7	17	3	47	13	3	23	7	3^2
41	71^2	53	3	7	-	3	-	-	3^2	13	7	23	79^2	17	19	31	29	3^2	-	11
43	3	37	7	13	-	23	3^2	-	-	7	-	-	3	-	17	3^2	7	11	3	53
45	5	7	5	5	3^2	5	5	3.5	7	29	3.5	5	5	3^2	5	7	3.5	19	5	3.5
47	7	-	3^2	-	13	3	-	7	3	19	-	3^2	-	11	7	-	17	13	41	-
49	3^2	19	29	3	-	31	7	-	-	3^2	23	11	3	7	-	37	61	17	3^2	-
51	-	17	59	-	23	7	-	3^2	-	11	3	-	7	29	-	-	3^2	43	17	7
53	31	-	17	53	7	3^2	-	11	3	-	-	7	13	-	3^2	-	-	3	7	17
55	35	5	5	3^2	5	11	3.5	5	5	3.5	7	5	3^2	31	5	3.5	5	7	3.5	13
57	13	3^2	7	11	17	-	-	19	-	7	3^2	47	-	13	11	79	7	29	-	3^2
59	-	7	3	23	53	17	-	13	31	59	73	3	11	-	3	7	-	3^2	19^2	-
61	7	13	-	3	43	67	3^2	7	-	3	11	61	3	-	7	3^2	-	-	3	-
63	61	3	19	31	3^2	-	7	17	11	67	43	-	-	3^2	23	-	3	-	-	11
65	13	5	3^2	29	5	3.5	11	5	3.5	5	5	3^2	7	19	3.5	13	31	3.5	5	7
67	3^2	-	23	3	7	19	3	73	-	3^2	-	7	3	-	29	11	59	67	3^2	-
69	37	3	11	7	3	-	-	3^2	-	47	7	31	-	11	-	-	3^2	7	-	23
71	11	-	7	41	-	3^2	53	29	19	7	13	11	-	23	3^2	-	7	37	-	-
73	19	7	-	3^2	13	-	31	23	7	11	-	-	3^2	-	-	7	-	13	29	19
75	5^2	3^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	-	31	3	19	-	11	7	53	3^2	43	59	29	-	7	17	-	11	3^2	23	-
79	3	-	-	11	-	7	3^2	-	-	3	-	37	7	-	11	3^2	-	-	3	7
81	-	11	-	—	3^2	-	13	41	-	-	3	7	11	3^2	-	-	17	-	7	13
83	13	71	3^2	7	-	3	-	-	37	31	7	3^2	61	13	3	29	41	7	-	-
85	3^2	17	7	3.5	5	5	3.5	13	11	3^2	5	5	3.5	5	5	3.5	7	23	3^2	11
87	-	7	17	-	31	37	11	3^2	7	-	3	23	-	3	13	7	3^2	11	71	17
89	7	-	41	17	11	3^2	-	7	13	53	-	3	19	-	3^2	11	-	31	83^2	29
91	3	29	11	3^2	17	-	7	-	43	3	-	41	3^2	7	-	13	-	-	3	-
93	11	3^2	67	-	3	7	-	3	71	13	3^2	11	7	3	43	19	23	-	61	3^2
95	5	5	3.5	13	7	3.5	17	19	3^2	11	23	3.5	5	5	3.5	5	13	3^2	7	5
97	3	-	-	7	23	29	3^2	17	-	3	7	-	3	-	73	3^2	37	7	11	-
99	-	3	7	-	3^2	11	41	3	17	7	19	-	-	3^2	67	-	7	13	-	3

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89
01	-	3^2	19	7	3	13	11	17	29	-	3^2	-	59	3	31	-	47	7.11	13	3^2
03	47	-	3	67	11	41	-	-	3^2	7	53	37	13	19	3	11	7	3^2	-	29
05	3.5	5.7	11	3.5	5	19	3^2	23	5.7	3.5	5	5	3.5	11	5	3^2	5	5	3.5	13
07	13	23	-	-	3^2	-	-	3.7	37	-	17	67	29	3^2	7	47	19	-	-	3
09	43	-	3^2	-	31	3	7	13	19	11	-	3^2	-	7	3	67	-	3	23	59
11	3^2	13	-	3	-	29	43	11	73	3^2	-	-	3.7	-	13	3	73	31	3^2	67
13	-	3	-	71	3.7	11	23	3^2	13	41	3	19	43	17	47	-	3^2	-	7	3
15	23	5	3.5	5.7	5	3^2	5	5	3.5	5	5.7	3.5	31	5	3^2	13	5	3.5	41	5
17	3.	11	7	3	-	-	3	-	-	3.7	-	-	3^2	-	19	17	7	23	3	37
19	-	3^2	-	13	3	73	19	31	7	-	3^2	23	-	47	-	7	13	-	-	3^2
21	17	-	29	-	41	23	-	7	3^2	89^2	13	3	-	53	3.7	-	37	3^2	-	11
23	3	17	31	3	13	-	3^2	-	-	19	71	-	3	29	-	3^2	-	13	17	-
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	-	-	3^2	17	7	13	29	-	3	-	23	3^2	19	11	3	-	-	3	13	79
29	3^2	-	-	3.7	17	-	3	59	-	3^2	31	11	13	-	-	3	-	29	3^2	-
31	79	3	7	-	3	17	13	3^2	41	7.11	3	47	-	3	-	19	3^2	-	-	13
33	13	7	3	-	-	3^2	17	19	3.7	-	29	3	-	13	3^2	23	89	41	73	-
35	3.5	5	5	3^2	5	11	3.5	5.7	5	3.5	5	5	3^2	5	5.7	3.5	11	5	3.5	5
37	31	3^2	-	23	37	-	7	3	17	-	3^2	79	-	3.7	13	-	3	-	-	3^2
39	-	59	19	41	43	3.7	-	71	3^2	17	-	3	7	31	29	-	53	3^2	-	7
41	3	37	13	3	7	-	3^2	-	-	3	17	7	41	19	23	3^2	-	-	3.7	-
43	-	3	-	7	3^2	19	-	29	23	13	3.7	17	-	3^2	-	-	43	7	37	11
45	5	5	3^2	13	5	3.5	11	5	3.5	5.7	5	3^2	17	5	3.5	5	5.7	11	29	5
47	3^2	7	-	31	11	-	3	61	19	3^2	13	-	3	17	-	11	-	-	3^2	23
49	19	3	11	-	13	-	-	3^2	47	-	3	29	73	11	17	83	3^2	13	-	19
51	11	-	3	-	-	3^2	7	23	3	-	83	11	37	7	3^2	17	41	3	53	-
53	3	23	-	3^2	29	13	3	-	-	11	-	31	3^2	-	79	3	17	-	13	7
55	17	3^2	5	5	3.5	5	5	11	5	37	3^2	5.7	13	3.5	19	29	3.5	17	5.7	3^2
57	-	17	41	7	-	11	13	-	3^2	73	7	3	23	61	3	43	11	3^2	17	13
59	13	-	17	11	-	-	3^2	-	29	3.7	-	41	3	13	11	3^2	7	19	3	17
61	23	11	53	17	3^2	-	47	13	7	19	3	-	11	3^2	-	7	3	-	-	29
63	7	13	3^2	37	17	3	79	7	3	-	11	3^2	-	-	3.7	-	-	23	-	-
65	3^2	5	5	3.5	5	17	3.5	5	11	3^2	5	23	3.5	5.7	5	3.5	5	5	3^2	11
67	37	3	13	53	19	23	17	3^2	-	31	3	-	7	3	-	13	3^2	11	-	3.7
69	-	67	3	-	7	3^2	-	17	43	13	-	3.7	-	-	3^2	41	-	37	7	-
71	3	71	11	3^2	31	67	3	19	17	3	7	-	3^2	11	43	3	13	7	3	-
73	11	3^2	7	73	47	-	-	3	-	17	3^2	11	-	3	37	-	3.7	31	19	3^2
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	3^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	3.7	-	19	3	-	-	3^2	7.11	-	3	41	13	31	-	7	3^2	-	67	11	47
79	-	3	29	47	3^2	13	7	3	-	79	3	-	17	3^2	61	23	11	-	13	41
81	73	43	3^2	61	-	3.7	-	31	37	23	-	3^2	13	17	11	-	-	3	83	7
83	3^2	11	-	23	7	-	13	43	-	3^2	59	7	11	83	17	3	19	-	3^2	13
85	13	3.5	31	5.7	3.5	31	29	3^2	19	5	11	5	5	3.5	5	17	3^2	5.7	5	3.5
87	19	-	3.7	83	-	3^2	-	13	11	7	-	3	-	-	3^2	31	17	29	-	43
89	17	13	37	3^2	-	-	11	-	23	3	-	19	3^2	-	13	3.7	-	17	3	89
91	7	3^2	23	19	11	-	-	3.7	13	61	3^2	-	-	3	7	71	3	59	17	3^2
93	41	-	11	-	59	3	7	-	3^2	-	-	3	-	7	19	13	-	3^2	-	17
95	11	5	5	3.5	5	5.7	3^2	5	5	3.5	5	11	3.5	23	5	3^2	37	5	3.5	5.7
97	47	3	-	13	3^2	71	43	23	53	11	3	7	-	3^2	29	-	13	19	31	3
99	31	23	3^2	7	-	17	-	11	3	-	13	3^2	43	37	3	-	-	3.7	11	-

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109
01	-	19	3	71	17	3	-	89	11	-	73	3.7	101^2	-	3	-	-	29	7	11
03	3	-	-	3.7	-	13	11	31	-	3	7	-	19	-	101	3^3	23	11	13	-
05	5	3.5	5.7	5	3.5	5	17	3.5	37	5.7	3.5	43	13	3.5	5	11	3.5	5	5	3.5
07	-	7	11	41	23	3	13	17	3.7	-	-	3^2	59	11	3	19	-	43	101	13
09	11	-	-	23	97^2	37	3	19	17	3^3	-	11	41	13	7	31	103^2	-	3^2	-
11	-	3	61	-	3	-	7	13	-	17	47	-	-	3.7	29	23	3^4	-	19	3
13	-	13	37	67	-	3.7	-	11	3	23	17	3	7	-	13	-	-	3	11	7
15	3.5	5	19	3.5	5.7	11	3.5	29	13	3.5	5	5.7	3.5	5	5	3.5	11	5	3.5	37
17	71	3^2	13	7	43	31	59	41	-	47	3.7	67	17	19	11	13	3	7	29	3^2
19	29	11	3.7	-	-	19	-	-	3^2	13	43	3	11	17	23	67	37	3^3	31	61
21	31	7	-	13	-	-	3^2	-	23	3	11	29	3	-	17	3.7	13	71	3	67
23	7	3	23	-	3^3	89	-	3.7	19	-	13	53	-	31	7	17	3	-	79	11
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	17	-	-	3	11	7	3	71	31	3^2	37	13	3.7	23	-	11	-	17	3^3	7
29	-	17	11	19	3.7	13	-	23	-	-	3	7	53	11	-	-	3^2	-	13	3
31	11	23	17	31	-	3^3	-	37	29	-	7	11	13	-	19	-	-	3.7	-	17
33	3	-	7	17	-	-	13	-	-	3.7	79	-	3^3	-	-	3	31	-	23	13
35	13	3.5	5	5	3.5	5	41	3.5	5.7	5	3.5	5	23	3.5	5	5.7	3.5	19	11	3.5
37	7	-	3	-	-	11	23	13	3^2	19	-	31	29	-	3.7	41	11	3^2	-	-
39	23	19	-	11	-	-	3.7	-	-	3	-	-	3	7	13	3^2	-	-	3	-
41	-	11	-	-	3^2	29	31	17	13	-	3	-	19	3^3	53	83	3	23	37	3.7
43	-	41	13	-	19	3	-	-	17	61	83	3.7	-	-	3	13	29	3	7	31
45	3.5	31	5	3.5	5	23	3.5	5	11	3.5	5.7	5	3.5	5	5	3.5	5	5.7	3.5	11
47	83	3	7	13	47	-	11	19	43	29	17	73	-	3	31	53	3.7	11	-	41
49	-	7	3	-	11	3^2	-	-	3.7	-	13	17	37	79	3^5	11	23	3	19	-
51	3.7	-	29	3^2	13	-	3	7	-	31	19	-	17	11	7	3	-	13	3	47
53	11	3^4	19	47	23	41	7	3	59	37	3^2	13	-	3.7	-	61	53	-	-	3^2
55	5	5	3.5	5	31	3.5	5	5	3.5	11	5	3.5	5.7	19	3.5	5	5	3.5	13	5.7
57	3	-	-	3	7	19	29	11	-	3	89	7	13	-	-	17	-	31	3.7	-
59	-	43	47	7	3^2	79	13	3	-	23	3.7	-	-	3^2	-	-	11	29	-	13
61	13	-	3.7	23	-	3	-	43	19	7	-	3^2	31	13	11	59	7	17	-	97
63	19	17	59	3	-	73	3	13	7	3^5	29	-	11	43	-	3.7	-	47	17	19
65	5.7	3.5	17	5	3.5	5	5	3.5	5	5	11	19	5	3.5	5.7	5	3.5	5	41	17
67	-	89	3	17	-	3^2	7	-	11	-	-	3	-	7	3^2	-	-	37	-	11
69	3	53	13	3^3	17	7	11	-	71	3	-	-	3.7	-	19	13	47	89	3	7
71	47	3^2	73	-	11	17	19	3	-	13	3^3	7	-	3	37	31	3	-	7	23
73	43	-	11	13	-	3	17	29	3^2	-	7	3	-	23	3	97	13	3.7	83	-
75	5^2	5^2	5^2	3^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	29	3.7	-	-	13	61	-	3	17	11	3	-	43	3^2	-	7	3	13	73	3
79	7	67	3^2	83	-	31	-	7.11	37	17	-	13	19	97	3.7	71	59	3	23	-
81	3^2	-	-	53	19	13	3.7	-	41	3^2	17	-	23	7	47	3	11	-	13	79
83	31	3	-	11	29	37	23	3^2	-	67	3	17	13	3	11	19	3^2	41	-	3.7
85	23	11	3.5	5	5.7	3.5	13	19	3.5	5	5	3.5	11	31	3.5	29	5	3.5	5.7	13
87	13	-	37	3.7	53	-	3	-	-	3	11	61	3^4	13	-	3	-	23	19	-
89	61	3^2	7	41	3	43	-	13	29	7	19	23	-	3	17	-	3.7	-	-	11
91	-	13	19	-	-	23	11	-	3.7	97	-	43	41	-	13	17	-	11	-	29
93	3.7	29	-	31	11	53	3^3	7	13	3	-	-	47	19	7	11	17	43	3	-
95	17	3.5	11	5	3.5	19	5.7	3.5	5	5	3.5	5	29	3.5	5	13	3.5	17	5	3.5
97	11	17	3^2	-	-	3.7	-	97	3	13	23	11	7	37	3	-	19	59	17	7
99	3^3	-	17	13	23	29	53	41	19	11	-	31	3	-	-	3	13	-	3.7	17

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126
01	19	17	23	3	13	31	3^2	-	-	3	11	-	3.7	-	-	3^3	-
03	-	3	17	89	3.7	-	41	47	29	-	3	13	-	3^2	79	-	3
05	31	5	3.5	5.7	5	3.5	5.11	5	3.5	5	5.7	3.5	5	23	3.5	41	5
07	3^2	29	7	3	17	37	53	23	-	3.7	-	-	13	31	19	3.11	7
09	101	3.7	11	43	3	17	13	3^2	7	-	3	-	29	3.11	-	7	3^2
11	13	41	37	-	-	3^2	17	7	31	43	-	3.11	-	13	3.7	-	-
13	3	-	-	3^2	101	29	3.7	13	-	3.11	41	-	23	7	-	43	-
15	5	3.5	5	31	3.5	5.7	23	3.5	17	5	3.5	5	5.7	3.5	13	5	3.5
17	23	-	3	-	7	349	-	-	13	17	61	3.7	19	109	3	-	31
19	3	-	13	3.7	19	-	3^2	-	53	29	17	-	3	97	11	13	-
21	103	337	7	-	3^3	41	-	3	-	13	3	17	101	3^2	-	19	3.7
23	73	7	29	13	-	23	59	19	3.7	-	11	3^3	17	-	41	7	13
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	-	3	103	47	13	-	7.11	3^2	-	-	19	67	-	3.7	17	-	23
29	41	31	19	-	11	3.7	29	37	3	79	23	13	7	-	3^2	17	73
31	3	-	11	3^2	23	13	3	-	-	41	53	7	3^3	19	31	3	17
33	17	3^2	47	7	37	19	-	3	-	-	3.7	11	13	3	-	83	3
35	5	17	3.5	5	5	3.5	13	5	3.5	5.7	29	3.5	5	5	3.5	23	5.7
37	13	37	17	3	-	83	3^3	97	19	23	-	53	3	13	-	3.7	-
39	19	47	-	23	31	11	103	3.7	-	-	3	61	-	3^3	7	-	3.11
41	61	13	3^2	11	17	3	7	59	3	-	-	19	-	41	3.11	-	-
43	3^3	11	-	19	-	97	3	-	13	3^2	-	-	3.7	-	23	37	47
45	5	3.5	13	5	3.5	5	17	3.5	23	5	3.5	5.7	31	3.5	19	13	3.5
47	-	71	23	7	-	3^2	19	17	359	13	7	3	37	-	3^3	-	-
49	29	-	7	13	107^2	-	353	31	41	3.7	-	-	3^2	53	59	47	13
51	43	3.7	-	-	149	-	61	3	7	19	13	29	-	23	-	7	3
53	7	19	31	-	13	3	43	23	3^3	-	17	3	-	11	3.7	-	-
55	3.5	23	5	3.5	29	5	3.5	5	5	3.5	5	5.11	3.5	5.7	47	3.5	5
57	-	3	-	41	19	13	-	3	71	11	3	-	103	3^2	-	29	3
59	-	-	3^3	37	7	3	89	11	59	-	31	3.7	13	17	3	19	-
61	3^2	-	-	3.7	73	11	13	19	29	3^3	7	-	61	47	17	53	11
63	13	3	7	11	3	31	107	3^2	-	7	3	-	-	13	103	17	3.7
65	5	5.7	3.5	5	5	3.5	5	13	3.5	5	19	3.5	5.11	5	3.5	5.7	17
67	3.7	13	19	3^3	-	43	3	41	-	3	11	23^2	29	83	13	59	53
69	-	17	59	-	3	23	7	3	13	-	3^3	43	-	3.7	37	-	41
71	-	-	13	83	-	3.7	11	79	3^2	-	-	3	7	89	3	13	-
73	3	-	-	17	7.11	71	3^2	61	31	13	-	37	3	-	-	3.11	19
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	19	-	3.7	31	23	17	-	-	37	29	13	3.11	-	-	3	-	7
79	3^2	7	-	3	13	-	17	-	7	3.11	47	19	3	-	-	3.7	31
81	7	3	29	19	43	37	-	3.7	109^2	-	3	13	-	3	7	23	3^2
83	-	53	3	-	-	13	7	-	17	23	43	31	71	29	19	-	11
85	3.5	5	37	3.5	5	5.7	3.5	5	5	17	5	5	3.5	5	5.11	3.5	43
87	-	113	-	59	3.7	-	13	3	-	-	17	7	11	3	-	41	3
89	13	67	53	7	-	3	-	-	3^2	19	7.11	17	-	13	23	-	-
91	3	19	7	3	-	67	3^3	13	23	3.7	107	73	17	-	-	3^2	37
93	-	3.7	23	-	3^2	-	11	3	7	67	29	89	19	17	13	7	3
95	5.7	5	3.5	43	5.11	3.5	5	5.7	3.5	5	41	3.5	5	37	17	5.11	5
97	3^3	-	13	29	-	-	3.7	47	-	31	-	-	3	23	-	13	-
99	11	3	-	-	3	7	-	19	73	13	37	11	7	3	29	43	17

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
01	13	17	19	–	3.11	43	47	3^2	23	29	3	37	–	3.13	59	11	3.7
03	–	31	3.11	–	–	3^3	53	13	3.7	61	71	43	–	19	3^2	7	–
05	3.5	13	29	3.5	5	19	3.5	5.7	37	3.5	5	5.11	3.5	5	5.7	3.5	5
07	97	3^2	–	–	17	47	7	41	13	11	3^2	–	–	3.7	–	–	19
09	71	–	13	–	–	3.7	–	23	19	31	–	3	7	–	3	13	41
11	19	23	–	3	7	11	17	–	59	3.13	–	7	3	–	103	3^2	11
13	–	3	37	7.11	31	73	–	17	–	–	3.7	19	–	3^3	11	61	3.13
15	5	5.11	3.5	19	43	3.5	5	5	3.5	5.7	5.13	3.5	5.11	5	3.5	5	5.7
17	3^3	7	–	3	13	–	23	–	7	17	29	41	3	107	19	3.7	103
19	23	3	–	47	3	–	19	3.7	11	–	17	13	31	3	7	59	37
21	–	–	59	29	–	3.13	7.11	–	3	53	–	17	–	7	3^3	–	–
23	3	–	–	3^2	11	7	3	31	–	19	–	23	3.7	37	29	3.11	–
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	13	101	31	7	–	3	–	29	3^3	–	37	3.11	19	83	17	41	–
29	3	–	7	43	19	–	3^2	13	83	3.7	–	–	3	–	71	3^3	23
31	29	3.7	67	83	3^2	101	–	3.11	7	43	23	–	–	3^2	13	19	17
33	17	41	3^3	–	23	3.11	67	19	3.13	–	31	29	–	–	3.7	43	11
35	3.5	17	13	3.5	37	5	3.5	5	5	3.5	41	5	3.5	5.7	5.11	3.5	47
37	47	3.11	17	–	29	31	–	3^2	–	13	19	101	7.11	3	67	23	3^3
39	–	37	19	17	7	3^2	–	89	3	23	11	3.7	53	101	3^2	29	13
41	31	–	–	3.7	17	–	3	–	11	3	7.13	–	3^2	19	79	47	–
43	–	3^2	43	–	3.13	17	11	3	29	7	3^3	109	73	31	–	–	3.7
45	5	5.7	3.5	5	5.11	3.5	17	5	3.5	5	5	3.5	5	5	3.5	5.7	19
47	3.7	29	107	3	–	13	3^2	17	19	3	59	61	3	11	43	3^2	–
49	61	3	23	–	3^3	–	7	3	17	–	3	11	29	3.7	–	–	3
51	41	71	3^2	31	–	3.7	79	–	3	17	–	3^3	7	–	53	–	113
53	13	–	–	19	7	29	3	11	–	31	17	7	3	23	–	3	31
55	5	3.5	5	5.7	3.5	5.11	5	3.5	5	5	3.5	17	5	3.5	19	5	3.5
57	–	13	3.7	11	59	3^3	19	–	3	7	–	31	17	–	3.11	53	7
59	3	7	–	3^2	–	–	61	43	7.13	29	–	–	3.11	17	–	3.7	83
61	7	3^2	13	37	41	89	31	3.7	71	19	3.11	83	23	43	17	13	3
63	–	19	29	–	–	3	23	–	3.11	13	–	3	–	41	3	17	53
65	3.5	31	5	3.5	5	5.7	3.5	5	5	3.5	5	47	3.5	29	5	3.5	5.13
67	17	3	–	73	3.7	–	–	3	–	79	3.13	7	–	3^3	31	11	3
69	113^2	17	3.11	7	13	3	29	–	3	–	7	23	61	11	3	19	–
71	3.11	61	109	3	–	23	3	19	41	3.7	47	97	3	–	37	67	7
73	53	3.7	–	17	3	13	43	3^3	7	113	3	–	89	3	–	7	3^2
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	3	79	19	3^2	–	17	3.7	–	–	47	23	–	3^2	7	–	3	11
79	13	3^3	–	11	23	7	17	3	37	–	3^2	–	7	19	11	109	3
81	–	11	3	103	7	19	–	17	3^3	–	–	3.7	31	–	29	–	73
83	3	13	–	3.7	–	37	3^2	97	17	3	7.11	–	59	–	13	3^3	19
85	5	3.5	5.7	5	3.5	5	5	3.5	5.11	5.7	3.5	5	5	3.5	5	5	3.5
87	19	7	3^3	23	–	43	11	–	3.7	–	17	3^2	71	–	3	7.13	–
89	3.7	–	31	3	109	97	3	41	107	3^3	–	17	3	73	7	3.11	–
91	–	3	11	19	3	–	7	3^2	–	–	3	29	17	3.7	23	31	3^3
93	11	–	61	–	79	3.7	59	103	23	–	13	3.11	7	17	19	–	37
95	3.5	5	23	3.5	5.7	5	3.5	5	5	3.5	31	5.7	3.5	5	17	3.5	5
97	67	3^2	41	7	53	–	–	3.11	–	–	3.7	13	–	37	–	17	3
99	–	–	3.7	–	67	3.11	–	–	3^2	19	–	41	–	23	3	79	17

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160
01	-	17	31	61	19	3	7	-	3^3	107	-	3	-	7	23	-	-
03	3	-	17	3.13	113	7	3^2	11	23	3	73	37	3.7	41	-	3^3	13
05	43	3.5	23	17	3.5	5.11	5	19	5	5	3.13	5.7	5	$3^2.5$	29	5	5.11
07	-	89	3^3	7.11	17	3	43	-	37	-	31	3^2	-	113	3.11	-	-
09	3^2	11	7	3	59	17	3	29	67	3^2	19	13	3.11	23	-	3	7
11	-	3.7	19	47	3	31	17	23	41	61	3.11	-	67	3	97	7	3^3
13	29	23	3	-	-	3^2	-	17	3.11	-	-	3	13	19	3.7	-	67
15	3.5	5	37	3.5	5	19	5.7	5	17	3.5	5	29	$3^2.5$	5.7	5	3.5	5
17	13	3^2	47	-	3.11	7	-	3	-	17	3^2	18	23	3.13	-	11	19
19	-	-	3.11	41	73	3	23	13	19	-	17	3.7	-	11	3	-	83
21	3.11	13	-	3.7	-	43	3^2	-	31	3	7	17	41	79	13	29	37
23	-	47	7	-	3^2	-	83	3	13	7.11	53	19	17	3^2	-	-	3.7
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^6	5^2	5^2	5^2	5^2
27	3.7	73	-	3	-	23	3	7	-	3.13	-	-	3	-	17	3	31
29	47	29	-	103	3	-	19	3^2	97	-	37	53	-	3.7	11	17	3.13
31	-	11	3	-	-	3.7	-	-	3	-	13	31	29	-	3^2	89	17
33	17	-	-	3^2	7.13	109	3	37	-	19	23	7	3^3	-	71	47	-
35	5	3.5	5	5.7	3.5	29	31	3.5	5.11	5	5.7	5.13	53	3.5	5	5	3.5
37	-	-	3.7	-	37	3.13	11	-	3^2	7	43	3	19	-	3	-	29
39	3	31	-	17	19	-	3^2	-	7	3	-	41	3.13	-	47	3.7	43
41	7	37	11^4	-	17	67	89	3.7	-	23	3	-	-	3^3	31	19	3
43	101	-	3^2	23	-	17	7	19	3	67	-	3.11	-	7.13	3	107	61
45	3.5	5	29	3.5	5	5.7	17	5.13	5	5.11	5	5	5.7	47	5	3.5	5
47	-	3.13	97	-	3.7	-	41	3^3	79	103	19	7	-	29	23	37	3^2
49	-	-	19	43	31	3.11	101	-	3.13	-	7	71	-	-	3^3	41	11
51	3	-	23	3.11	-	-	29	109	101	3.7	-	-	37	19	11	3.13	7
53	97	3.7	-	-	3	19	-	3	7	13	17	103	11	59	83	43	3
55	5.7	41	3.5	5.13	5	3.5	5	5.7	$3^2.5$	37	5.11	17	31	23	5.7	5	5.13
57	61	-	-	3	83	-	3.7	23	19	3	29	47	17	7	101	3^2	-
59	19	23	107	-	3.13	7	37	31	-	-	3	-	7	17	-	-	53
61	-	-	3^3	29	7	3	-	-	3	-	-	3.7	-	-	17	11	-
63	3^2	-	31	3.7	89	13	3	59	-	3^3	47	79	23	11	29	17	-
65	5.11	3.5	5.7	5	3.5	41	23	$3^2.5$	43	5.7	3.5	5.11	5.13	3.5	19	31	5.7
67	17	7	3	-	-	3^2	19	29	3.7	11	-	3	-	-	41	7	-
69	3.7	17	-	3^3	-	-	3	7.11	-	47	31	-	3^2	13	7	3	-
71	29	3^2	17	-	3	11	7	3.13	-	19	3^3	23	-	3.7	59	-	3.11
73	41	19	67	17	107	3.7	-	-	3^2	-	-	29	7	-	3.11	-	-
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	31	43	13	7	3^3	17	-	3	-	-	3.7	37	61	3^2	-	13	23
79	-	61	3.7	-	-	3	17	43	3.11	7.13	23	3^3	-	31	67	19	7
81	3^2	7	53	3.13	23	71	3.11	17	37	3^2	113	-	3	43	-	3.7	13
83	7	3	-	-	3.11	-	-	3.7	17	-	3.13	-	-	3	7	11	3^2
85	5	5	3.5	5	5.13	3.5	5.7	5	3.5	17	19	3.5	5	5.7	$3^2.5$	23	5
87	3.11	29	19	31	-	7	47	-	-	23	17	109	3^3	-	-	3	-
89	-	3^2	37	23	3.7	13	79	61	-	11	3^2	17	29	19	-	59	31
91	43	-	59	7	-	19	-	11	3^2	-	7	3	17	-	3	-	-
93	3	-	7	3	53	29	3^3	-	41	3.7	-	31	3	17	23	3^2	19
95	5.13	3.5	5	5.11	3.5	5	5	3.5	5.7	5	3.5	5	43	5.13	5.11	5.7	29
97	19	11	23	-	-	3	31	7.13	3	89	-	3^2	11	-	3.7	17	-
99	3^3	13	-	3	47	53	3.7	-	-	29	11	19	3	37	13	3	17

	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177
01	3^2	17	-	3.7	29	13	19	53	-	3^2	7	103	73	-	37	3	31
03	-	3.11	17	47	3	-	-	3^2	-	7	3	-	11.13	3	23	29	3.7
05	5	5.7	3.5	17	5	$3^2.5$	5.13	5	5.7	19	5.11	31	5	5	$3^2.5$	5.7	5
07	3.7	19	23	3^2	17	-	3	7^5	29	3	-	-	3^2	103	41	3	-
09	89	3^2	47	61	3	17	31	3.13	37	73	3^2	-	19	3.7	-	-	3
11	-	29	3	-	19	3.7	17	-	3^2	-	71	3	7	23	3.13	11	89
13	41	31	11	3	7	37	3^3	17	13	53	109	7	29	11	83	19	-
15	5.11	23	5.13	5.7	$3^2.5$	5	5	19	17	41	5.7	5.11	5	$3^2.5$	31	5.13	3.5
17	71	-	3.7	-	83	29	73	67	3	7.11	-	3^2	-	-	3^2	79	7
19	3^3	7	-	3.13	-	-	3	11	7	31	19	67	23	-	-	3.7	47
21	47	3	19	-	3	11	23	3^3	-	-	3.13	17	-	3	7	67	3.11
23	23	-	3	11	31	3^2	7	-	3	29	-	3	17	19	3^3	-	37
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	-	3^3	29	-	3.7	13	43	71	-	-	3.11	23	-	37	17	-	19
29	127^2	-	3	7	-	23	-	-	3^3	-	7	3	31	29	3	17	-
31	19	-	7	3	61	-	3.11	-	-	3.7	37	-	53	-	47	3^3	7.17
33	17	3.7	-	-	3.11	-	29	31	41	-	3	19	-	3.13	89	7.11	23
35	5.7	17	5.11	19	5	3.5	5	5.7	3.5	5	23	$3^2.5$	5	5.11	5.7	5	5
37	3.11	13	17	3	23	127	3.7	113	-	3^3	-	11	3	47	19	3	-
39	-	3	-	17	37	7	19	3^2	13	11	29	-	7	3	-	31	3^3
41	-	109	3.13	41	17	3^2	-	11	3	-	61	3.7	-	107	3^2	23	113
43	3	37	59	3^3	71	17	3	-	-	3.13	31	43	41	-	53	3	11
45	5	19	5.7	5.11	3.5	5	17	3.5	5	5.7	$3^2.5$	5	5	3.5	5.11	5	5.7
47	67	7.11	3	-	-	31	-	17	3.7	-	13	3	19	73	3	7	-
49	3.7	-	-	3	19	-	3^2	29	17	3	11	47	3	-	23	37	-
51	31	3	83	-	3^3	-	7	41	23	59	3	13	-	3.7	-	19	61
53	29	-	23	-	-	3.7	11	19	3	-	17	3^3	37	31	3	127	41
55	$3^2.5$	5	5	3.5	5.7	5	3.5	5	5	$3^2.5$	47	5.7	5.13	5	5	5.11	53
57	107	3	11	7	3	-	13	3^2	31	37	3.7	-	17	3.11	97	-	3^2
59	113	71	3.7	109	29	3^3	-	23	3	7	-	3.11	-	79	3^2	-	43
61	3	23	-	31	-	-	37	13	7	3.11	131^2	41	3^3	19	17	3.7	-
63	7	3.13	-	101	3	19	-	3.7	-	113	3^2	61	97	3	7.13	17	31
65	53	5	3.5	37	5	5.11	5.7	5	$3^2.5$	5	5	3.5	23	5.7	3.5	5	5.11
67	17	-	13	3.11	-	7	3^3	101	19	3	-	31	3.7	-	11	3.13	109
69	19	17	-	43	3.7	79	41	3	71	101	59	7	11	3^3	-	-	3
71	103	53	17	7.13	73	3	31	-	3	43	7.11	19	29	-	3	41	13
73	3^2	-	7	17	-	-	3	47	11	3.7	13	23	3	101	-	43	7
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	7	41	53	-	11	17	19	7	3	-	89	3.13	-	-	31	11	29
79	3	73	11	3^2	59	13	3.7	-	-	3	41	37	3^2	7.11	-	71	23
81	11	3^3	-	-	3	7	97	3.17	-	19	23	11	7.13	3	-	-	3
83	-	19	43	53	23	67	13	-	3^3	11	-	3.7	-	-	3	-	-
85	5.13	5	29	5.7	31	47	$3^2.5$	5.11	43	5.17	5.7	5	19	5.13	5	$3^2.5$	5
87	-	61	7	-	19	37	-	3.13	-	7	3.17	59	-	29	43	23	3.7
89	-	7.13	3^3	11	53	3	103	-	3.7	23	-	3.17	-	-	3.11	19	-
91	3.7	11	37	23	47	-	29	19	13	3^3	-	-	3.17	-	7	3	-
93	-	3	97	-	3	-	7	3^2	-	-	3.11	-	-	3.7	73	13	3^3
95	41	5	3.5	5	5	5.7	5	31	5.11	5.13	19	3.5	5.7	37	$3^2.5$	5	5
97	3	43	19	3^3	7	59	3.11	61	23	41	29	7	3^2	-	-	3.17	37
99	97	3^2	23	7	3.11	-	107	43	89	-	3^3	-	127	19	-	11	3.17

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193
01	7	3^3	47	23	3	-	-	3.7	19	-	3^2	41	-	3	7.13	-
03	19	-	3.17	43	109	3	7.11	-	3^3	59	-	3	31	7	37	97
05	3.5	5	13	3.17	5.11	5.7	$3^2.5$	5	5	29	5	19	5.7	5	23	5.11
07	-	47	11	19	3.7	-	79	31	23	13	3	37	83	3.11	-	43
09	11	-	3^3	7.13	131	3.17	41	83	3	53	7	3.11	-	97	3.19	-
11	3^2	-	31	3	-	-	3.17	107	37	3^3	13	-	3	29	-	41
13	47	3.7	-	59	3.13	-	-	3.11	7	-	3	-	-	23	-	31
15	5.7	5	3.5	5	5	5.11	29	5.7	5.17	19	37	5.13	5	5	5.7	5
17	3	19	43	3^3	-	13	3.7	-	-	3.17	31	-	3^2	7	11	47
19	29	3.11	37	-	3	7	113	3	43	-	3^3	-	7.13	3	-	-
21	71	-	3	-	19	31	109	-	3^2	97	59	3.7	23	-	43	139^2
23	3.13	-	67	3.7	-	73	23	-	11	3	7	127	3.17	13	47	3.19
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5	5^2	5^2	5^2
27	-	7.13	3^2	-	11	41	-	97	3.7	61	67	3^3	53	31	3.17	7.11
29	3.7	-	11	3	-	-	3	7	13	3^2	19	23	3	37	41	3.17
31	11	43	19	-	59	23	7	29	31	-	3	11	-	3.7	-	13
33	17	79	3	-	-	3^3	-	43	3	131	37	3	7	53	3^2	-
35	29	5.17	5	5.13	5.7	19	3.5	5.11	5	3.5	5	5.7	$3^2.5$	43	5	3.5
37	-	3^2	17	7	3	11	103	37	-	41	3.7	29	-	3	-	61
39	-	-	3.7	97	23	3	-	-	19	7	-	59	79	-	3.11	83
41	19	7.11	-	3	29	-	3^3	-	7	3	83	31	3.11	-	71	3.7
43	7	3	-	-	3^2	83	-	3.7	103	-	3.11	19	137	3^3	7	23
45	43	37	$3^2.5$	19	41	3.5	5.7	5	5.11	23	5	$3^2.5$	5.13	5.7	3.5	53
47	3^3	131	-	23	71	7	3.11	17	29	3^2	47	-	3.7	41	19	3
49	13	31	-	-	3.7	59	19	3^3	17	-	61	7	43	3.13	-	11
51	-	29	3.11	7	-	3^2	-	13	3	17	7	3	-	11	3^3	37
53	3.11	13	7	3^2	-	-	3	-	23	3.7	17	11	29	107	13	3
55	5	5.7	23	5	3.5	5	5	3.5	5.7	5.11	$3^2.5$	5.17	37	3.5	5	5.7
57	7	-	3.13	67	-	29	-	7.11	3^3	-	109	71	59	-	3.7	13
59	3	-	-	3	19	11	3.7	67	47	3.13	-	-	3	23	-	3^3
61	53	3	-	127	3^2	43	-	23	-	73	3	61	7	3^2	103	19
63	-	23	3^3	41	7	3	37	19	3	29	13	3.7	11	-	3	67
65	$3^2.5$	5	5	5.7	5.13	5	3.5	47	5	$3^2.5$	5.7	5	31	5	5	3.5
67	17	53	89	37	3	-	59	3^2	11	7	19	13	23	3	-	107
69	107	7.17	19	-	-	3.13	23	31	3.7	137^2	-	3	-	29	3^2	7
71	3.7	-	17	3^3	11	-	47	7	-	3	113	61	13	19	7	3.11
73	61	3^2	31	17	3	19	29	41	71	-	3^3	-	-	3.7	-	-
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	59	-	-	73	7	23	3^2	13	19	3.11	43	7	3	127	37	3^2
79	19	3.13	101	53	3^3	-	17	3.11	-	89	3.7	-	-	3^2	13	-
81	-	-	3.7	-	101	3.11	-	17	3.13	7	79	3^3	-	-	3	-
83	3^2	7	107	3.11	47	31	61	-	7.17	3^2	23	41	3	-	11	3.7
85	5.7	5.11	5	5	23	5	5	5.7	37	5.13	3.5	5	5.11	3.5	5.7	5
87	31	-	3	13	-	3^3	19	-	3	-	101	3	-	7	3^2	-
89	67	-	-	43	-	37	3	29	11	3	13	17	3^3	31	-	23
91	-	3^2	79	-	3.7	53	41	3	-	19	3^2	7	17	3	101	-
93	29	19	37	23	11	3	-	-	31	-	7	3.13	61	17	59	41
95	3.5	59	5.7	3.5	5	5.13	$3^2.5$	5	5	5.7	5	29	5.19	5.11	5.17	$3^2.5$
97	11	3.7	-	31	19	-	53	3	7	-	3	11	113	3^3	23	7.17
99	7	41	3^2	-	29	3	13	7	23	11	-	3^2	71	73	3.7	19

I. TABLE OF PRIME AND COMPOSITE NUMBERS.

	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209
01	29	-	17	3.11	-	7	59	-	-	67	23	83	3^3	127	31	3
03	-	3.11	-	61	3.7	13	83	3	89	79	3^2	29	11	67	71	-
05	5	47	3.5	5.7	5.17	3.5	5	5	3^2.5	31	5.7	3.5	5.13	41	5.19	37
07	3	-	7	3	29	17	3^3	-	11	3.7	-	-	3	-	-	23
09	13	3.7	-	-	31	43	107	3	7	23	3	-	37	3^3	-	29
11	47	109	3^2	23	11	3	-	7.13	3	19	-	43	-	139	3.7	11
13	3^3	79	11	3	-	-	3.7	-	41	37	137	73	3	7.11	13	3
15	5.11	3.5	5	5	3.5	5.7	5	3^3.5	5.13	5.17	3.5	5.11	5.7	3.5	23	47
17	-	29	3.13	-	7.19	3^2	37	-	23	11	17	3.7	53	-	3^3	13
19	3	131	23	3.7	-	-	3	31	-	3.13	7	71	29	-	109	3.19
21	-	3^3	7	37	3	11	-	3.19	73	7	3^2	-	17	3	47	-
23	-	7	31	11	43	29	-	-	3^3	-	13	3	41	23	3.11	61
25	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
27	-	23	19	-	3^2	-	7	3	113	-	3.11	13	-	3.7	59	17
29	-	59	3^3	109	79	3.7	-	-	3.11	29	31	3^2	7	19	53	-
31	3.17	-	67	3	7	19	3.11	41	-	3^3	-	7	23	-	37	3
33	-	3.17	29	7	3.11	31	23	3^2	-	-	3.7	-	47	3	83	11
35	5.13	5	5.7	5	5	3^2.5	5	5	5.19	5.7	61	3.5	5	5.11	3^2.5	53
37	3.11	7	73	3^3	83	-	3	13	59	3	107	11	3^2	89	67	3.7
39	7	3.13	41	-	3.17	127	29	3.7	37	43	3^3	23	-	31	7.13	-
41	-	-	3	19	-	3.17	7	11	3.13	-	-	41	-	7	3	43
43	3	-	13	3	-	37	3.17	-	31	3	-	-	3.7	-	19	3.13
45	5	3.5	5	5.11	5.7	5	5.19	5.17	5	5.13	29	5.7	5	3^2.5	5.11	59
47	-	11	37	31	89	61	-	-	3.17	-	23	3^3	11	-	3	-
49	3^2	113	7	29	23	-	41	-	-	3.7	11.13	-	3	-	-	3
51	53	3.7	43	-	3.13	71	-	3^2	7.11	47	3.17	-	107	3	29	41
53	7	-	3	-	-	3^3	11	7	43	-	119	3.17	19	-	3.7	23
55	3.5	5	5	3^2.5	5.11	5.13	5.7	29	5	23	5	5	5.17	5.7	43	5.11
57	-	41	11	23	3	7	31	3	47	-	3^2	61	7.13	3.17	-	19
59	29	-	3	-	7	3	13	19	3^2	-	41	3.7	73	-	3.17	-
61	3.13	31	-	3.7	-	-	3^3	-	-	3.11	37	29	71	13	23	3.17
63	-	3	53	-	3^2	-	-	3.11	23	7	3.19	-	-	3^3	31	-
65	5.17	5.7	5.19	59	29	5.11	5	37	5.7	5	5	3^2.5	5	5	5.13	5.7
67	3^3	17	71	3.11	-	41	3	43	13	31	97	131	3	19	7.11	29
69	-	3.11	89	53	37	19	47	3^3	-	-	3	67	11	3.7	41	13
71	-	-	79	17	31	3.7	-	23	29	13	11	3	7	-	3^3	67
73	3	23	103	3.13	7.17	-	3	-	97	3	59	7	3^2	-	-	3
75	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2	5^2
77	-	-	3.7	-	139	3	17	-	3^3	41	-	3.19	23	79	3	11
79	43	7	11	3.19	103	-	23	17	7	3	-	13	61	11	-	3^3
81	23	61	-	131	3^2	29	43	3.7	17	89	3	11	-	3^2	7.19	-
83	-	-	3^9	73	59	3	7.19	-	3	109	-	3^2	37	7	3	-
85	3^2	5	31	3.5	41	5.7	5.13	5.11	5	3^3.5	5.17	23	5.7	5	5	3.5
87	13	3	-	47	3.7	23	53	3^2	-	29	3	7.17	137	41	-	31
89	-	19	3	7.11	-	3^2	-	13	3	-	7	3	17	-	3.11	139
91	73	137	29	3^3	-	-	37	61	103	3.7	31	59	3.11	17	13	3
93	101	3.7	47	-	3.19	-	71	53	7.13	-	3^3	-	-	29	17	7
95	5.7	5	5.13	37	23	31	5	5.7	5.11	5	5	3.5	5	5	5.7	5.13
97	67	-	-	3	101	-	3.7	19	-	3.13	103	43	3	7	-	3^2
99	31	47	-	13	3^3	7	101	3	53	-	3	-	7	3^2	-	23

TABLE II.

Showing the amount of $1.00, at compound interest, from 1 year to 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	6 p. cent.	7 p. cent.
1	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.125509	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.159274	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.229874	1.272279	1.315932	1.360862	1.407100	1.503630	1.605781
8	1.266770	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.304773	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.343916	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.425761	1.511069	1.601032	1.695881	1.795856	2.012196	2.252192
13	1.468534	1.563956	1.665073	1.772196	1.885649	2.132928	2.409845
14	1.512590	1.618694	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.557967	1.675349	1.800943	1.935282	2.078928	2.396558	2.759031
16	1.604706	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.652848	1.794675	1.947900	2.113377	2.292018	2.692773	3.158815
18	1.702433	1.857489	2.025816	2.208479	2.406619	2.854339	3.379931
19	1.753506	1.922501	2.106849	2.307860	2.526950	3.025599	3.616526
20	1.806111	1.989789	2.191123	2.411714	2.653298	3.207135	3.869683
21	1.860295	2.059431	2.278768	2.520241	2.785963	3.399564	4.140561
22	1.916103	2.131512	2.369919	2.633652	2.925261	3.603537	4.430400
23	1.973586	2.206114	2.464715	2.752166	3.071524	3.819750	4.740528
24	2.032794	2.283328	2.563304	2.876014	3.225100	4.048935	5.072365
25	2.093778	2.363245	2.665836	3.005434	3.386355	4.291871	5.427431
26	2.156591	2.445959	2.772470	3.140679	3.555673	4.549383	5.807351
27	2.221289	2.531567	2.883369	3.282009	3.733456	4.822346	6.213866
28	2.287928	2.620177	2.998703	3.429700	3.920129	5.111687	6.648836
29	2.356565	2.711878	3.118651	3.584036	4.116136	5.418388	7.114255
30	2.427262	2.806794	3.243397	3.745318	4.321942	5.743491	7.612253
31	2.500080	2.905031	3.373133	3.913857	4.538039	6.088101	8.145110
32	2.575083	3.006708	3.508059	4.089981	4.764941	6.453387	8.715268
33	2.652335	3.111942	3.648381	4.274030	5.003188	6.840590	9.325337
34	2.731905	3.220860	3.794316	4.466361	5.253348	7.251025	9.978110
35	2.813862	3.333590	3.946089	4.667348	5.516015	7.686087	10.676578
36	2.898278	3.450266	4.103932	4.877378	5.791816	8.147252	11.423939
37	2.985227	3.571025	4.268090	5.096860	6.081407	8.636087	12.223614
38	3.074783	3.696011	4.438813	5.326219	6.385477	9.154252	13.079277
39	3.167027	3.825372	4.616366	5.565899	6.704751	9.703507	13.994827
40	3.262038	3.959260	4.801021	5.816364	7.039989	10.285718	14.974465
41	3.359899	4.097834	4.993061	6.078101	7.391988	10.902861	16.022677
42	3.460696	4.241258	5.192784	6.351615	7.761587	11.557033	17.144265
43	3.564517	4.389702	5.400495	6.637438	8.149667	12.250455	18.344363
44	3.671452	4.543342	5.616515	6.936123	8.557150	12.985482	19.628469
45	3.781596	4.702358	5.841176	7.248248	8.985008	13.764611	21.002461
46	3.895044	4.866941	6.074823	7.574420	9.434258	14.590487	22.472634
47	4.011895	5.037284	6.317816	7.915268	9.905971	15.465917	24.045718
48	4.132252	5.213589	6.570528	8.271455	10.401267	16.393872	25.728918
49	4.256219	5.396065	6.833349	8.643671	10.921333	17.377504	27.529943
50	4.383906	5.584927	7.106683	9.032636	11.467400	18.420154	29.457039

TABLE III.

THE AMOUNT OF AN ANNUITY OF $1.00 FROM 1 YEAR TO 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	5½ p. cent.	6 p. cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.030000	2.035000	2.040000	2.045000	2.050000	2.055000	2.060000
3	3.090900	3.106225	3.121600	3.137025	3.152500	3.168025	3.183600
4	4.183627	4.214943	4.246464	4.278191	4.310125	4.342266	4.374616
5	5.309136	5.362466	5.416322	5.470710	5.525631	5.581091	5.637093
6	6.468410	6.550152	6.632975	6.716892	6.801913	6.888051	6.975319
7	7.662462	7.779408	7.898294	8.019152	8.142008	8.266894	8.393838
8	8.892336	9.051687	9.214226	9.380014	9.549109	9.721573	9.897468
9	10.159106	10.368496	10.582795	10.802114	11.026564	11.256259	11.491316
10	11.463879	11.731393	12.006107	12.288210	12.577893	12.875354	13.180795
11	12.807796	13.141992	13.486351	13.841179	14.206787	14.583498	14.971643
12	14.192029	14.601962	15.025805	15.464032	15.917127	16.385590	16.869941
13	15.617790	16.113030	16.626838	17.159913	17.712983	18.286798	18.882138
14	17.086324	17.676986	18.291911	18.932109	19.598632	20.292572	21.015066
15	18.598914	19.295681	20.023588	20.784054	21.578564	22.408663	23.275971
16	20.156881	20 971030	21.824531	22.719337	23.657492	24.641139	25.672528
17	21.761588	22.705016	23.697512	24.741707	25.840366	26.996402	28.212880
18	23.414436	24.499691	25.645413	26.855084	28.132385	29.481205	30.905653
19	25.116868	26.357180	27.671229	29.063562	30.539004	32.102671	33.759992
20	26.870374	28.279682	29.778078	31.371423	33.065954	34.868318	36.785592
21	28.676486	30.269471	31.969202	33.783137	35.719252	37.786075	39.992727
22	30.536780	32.328902	34.247970	36.303378	38.505214	40.864309	43.392290
23	32.452884	34.460414	36.617888	38.937030	41.430475	44.111846	46.995828
24	34.426470	36.666528	39.082604	41.689196	44.501999	47.537998	50.815577
25	36.459264	38.949857	41.645908	44.565210	47.727099	51.152588	54.864512
26	38.553042	41.313102	44.311745	47.570645	51.113454	54.965979	59.156383
27	40.709634	43.759060	47.084214	50.711324	54.669126	58.989109	63.705766
28	42.930923	46.290627	49.967583	53.993333	58.402583	63.233510	68.528112
29	45.218850	48.910799	52.966286	57.423033	62.322712	67.711353	73.639798
30	47.575416	51.622677	56.084938	61.007070	66.438847	72.435478	79.058186
31	50.002678	54.429471	59.328335	64.752388	70.760790	77.419429	84.801677
32	52.502759	57.334502	62.701469	68.666245	75.298829	82.677498	90.889778
33	55.077841	60.341210	66.209527	72.756226	80.063770	88.224760	97.343165
34	57.730177	63.453152	69.857909	77.030256	85.066959	94.077122	104.183755
35	60.462082	66.674013	73.652225	81.496618	90.320307	100.251363	111.434780
36	63.275944	70.007603	77.598314	86.163966	95.836323	106.765188	119.120867
37	66.174223	73.457869	81.702246	91.041344	101.628139	113.637274	127.268119
38	69.159449	77.028895	85.970336	96.138205	107.709546	120.887324	135.904206
39	72.234233	80.724906	90.409150	101.464424	114.095023	128.536127	145.058458
40	75.401260	84.550278	95.025516	107.030323	120.799774	136.605614	154.761966
41	78.663296	88.509537	99.826536	112.846688	127.839763	145.118923	165.047684
42	82.023196	92.607371	104.819598	118.924789	135.231751	154.100464	175.950545
43	85.483892	96.848629	110.012382	125.276404	142.993339	163.575989	187.507577
44	89.048409	101.238331	115.412877	131.913842	151.143006	173.572669	199.758032
45	92.719861	105.781673	121.029392	138.849965	159.700156	184.119165	212.743514
46	96.501457	110.484031	126.870568	146.098214	168.685164	195.245720	226.508125
47	100.396501	115.350973	132.945390	153.672633	178.119422	206.984234	241.098612
48	104.408396	120.388257	139.263206	161.587902	188.025393	219.368367	256.564529
49	108.540648	125.601846	145.833734	169.859357	198.426663	232.433627	272.958401
50	112.796867	130.997910	152.667084	178.503028	209.347996	246.217477	290.335905

TABLE IV.

THE PRESENT WORTH OF AN ANNUITY OF $1.00 FROM 1 YEAR TO 50.

Year.	3 p. cent.	3½ p. cent.	4 p. cent.	4½ p. cent.	5 p. cent.	5½ p. cent.	6 p. cent.	Year.
1	0.97087	0.96618	0.96154	0.95694	0.95238	0.94786	0.94339	1
2	1.91347	1.89969	1.88609	1.87267	1.85941	1.84631	1.83339	2
3	2.82861	2.80164	2.77509	2.74896	2.72325	2.69793	2.67301	3
4	3.71710	3.67308	3.62990	3.58753	3.54595	3.50514	3.46511	4
5	4.57971	4.51505	4.45182	4.38998	4.32948	4.27028	4.21236	5
6	5.41719	5.32855	5.24214	5.15787	5.07569	4.99553	4.91732	6
7	6.23028	6.11454	6.00205	5.89270	5.78637	5.68297	5.58238	7
8	7.01969	6.87396	6.73274	6.59589	6.46321	6.33457	6.20979	8
9	7.78611	7.60769	7.43533	7.26879	7.10782	6.95220	6.80169	9
10	8.53020	8.31661	8.11090	7.91272	7.72173	7.53762	7.36009	10
11	9.25262	9.00155	8.76048	8.52892	8.30641	8.09254	7.88687	11
12	9.95400	9.66333	9.38507	9.11858	8.86325	8.61852	8.38384	12
13	10.63495	10.30274	9.68565	9.68285	9.39357	9.11708	8.85268	13
14	11.29607	10.92052	10.56312	10.22283	9.89864	9.58965	9.29498	14
15	11.93794	11.51741	11.11839	10.73955	10.37966	10.03759	9.71225	15
16	12.56110	12 09412	11.65230	11.23401	10.83777	10.46216	10.10589	16
17	13.16612	12.65132	12.16567	11.70719	11.27407	10.86461	10.47726	17
18	13.75351	13.18968	12.65930	12.15999	11.68959	11.24607	10 82760	18
19	14.32380	13.70984	13.13394	12.59329	12.08532	11.60765	11.15812	19
20	14.87747	14.21240	13.59033	13.00794	12.46221	11.95034	11.46992	20
21	15.41502	14.69797	14.02916	13.40472	12.82115	12.27524	11.76408	21
22	15.93692	15.16712	14.45112	13.78442	13.16300	12.58317	12.04158	22
23	16.44361	15.62041	14.85684	14.14777	13.48857	12.87504	12.30338	23
24	16.93554	16.05837	15.24696	14.49548	13.79864	13.15170	12.55036	24
25	17.41315	16.48151	15.62208	14.82821	14.09394	13.41391	12.78336	25
26	17.87684	16.89035	15.98277	15.14661	14.37518	13.66250	13.00317	26
27	18.32703	17.28536	16.32959	15.45130	14.64303	13.89810	13.21053	27
28	18.76411	17.66702	16.66306	15.74287	14.89813	14.12142	13.40616	28
29	19.18845	18.03577	16.98371	16.02189	15.14107	14.33310	13.59072	29
30	19.60044	18.39205	17.29203	16.28889	15.37245	14.53375	13.76483	30
31	20.00043	18.73628	17.58849	16.54439	15.59281	14.72393	13.92909	31
32	20.38877	19.06887	17.87355	16.78889	15.80268	14.90420	14.08404	32
33	20.76579	19.39021	18.14765	17.02286	16.00255	15.07507	14.23023	33
34	21.13184	19.70068	18.41120	17.24676	16.19290	15.23703	14.36814	34
35	21.48722	20.00066	18.66461	17.46101	16.37419	15.39055	14.49825	35
36	21.83225	20.29049	18.90828	17.66604	16.54685	15.53607	14.62099	36
37	22.16724	20.57053	19.14258	17.86224	16.71129	15.67400	14.73678	37
38	22.49246	20.84109	19.36786	18.04999	16.86789	15.80474	14.84602	38
39	22.80822	21.10250	19.58448	18.22965	17.01704	15.92866	14.94907	39
40	23.11477	21.35507	19.79277	18.40158	17.15909	16.04612	15.04630	40
41	23.41240	21.59910	19.99305	18.56611	17.29437	16.15746	15.13802	41
42	23.70136	21.83488	20.18563	18.72355	17.42321	16.26299	15.22454	42
43	23.98190	22.06269	20.37079	18.87421	17.54591	16.36303	15.30617	43
44	24.25427	22.28279	20.54884	19.01838	17.66277	16.45785	15.38318	44
45	24.51871	22.49545	20.72004	19.15635	17.77407	16.54772	15.45583	45
46	24.77545	22.70092	20.88465	19.28837	17.88007	16.63283	15.52437	46
47	25.02471	22.89943	21.04294	19.41471	17.98102	16.71357	15.58903	47
48	25.26671	23.09124	21.19513	19.53561	18.07716	19.79011	15.65003	48
49	25.50166	23.27656	21.34147	19.65130	18.16872	16.86266	15.70757	49
50	25.72976	23.45562	21.48218	19.76201	18.25593	16.93143	15.76186	50

V.

TABLE OF ROOTS AND POWERS.

1st power,	1	2	3	4	5	6	7	8	9
2d power,	1	4	9	16	25	36	49	64	81
3d power,	1	8	27	64	125	216	343	512	729
4th power,	1	16	81	256	625	1296	2401	4096	6561
5th power,	1	32	243	1024	3125	7776	16807	32768	59049
6th power,	1	64	729	4096	15625	46656	117649	262144	531441
7th power,	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th power,	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power,	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th power,	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th power,	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
12th power,	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
13th power,	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
14th power,	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
15th power,	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

VI.

TABLE FOR ASCERTAINING THE NUMBER OF DAYS FROM ANY DAY IN THE YEAR, TO ANY OTHER DAY.

1st Mo., Jan.	- 0	5th Mo., May	- 120	9th Mo., Sept.	243
2d Mo., Feb.	- 31	6th Mo., June	- 151	10th Mo., Oct.	273
3d Mo., Mar.	- 59	7th Mo., July	- 181	11th Mo., Nov.	304
4th Mo., Apl.	- 90	8th Mo., Aug.	- 212	12th Mo., Dec.	334

RULE.

To the given day of each month add the tabular number for the month, and subtract the less sum from the greater.

If the two dates are in different years, subtract the result thus found, from 365.

In leap years, add 1 *to the number after the* 28*th of February.*

EXAMPLE.

How many days from Feb. 13, to Oct. 29, 1844?

Feb. 13	Oct. 29	302 — 44 = 258
31	273	Add for leap year 1
44	302	259 Ans.

VII.

TABLES OF MONEY.

UNITED STATES.

10 mills (m.)	make 1 cent.	ct.
10 cents	make 1 dime.	
10 dimes	make 1 dollar.	$
10 dollars	make 1 eagle.	

GREAT BRITAIN.

4 farthings (qr.)	make 1 penny.	.
12 pence	make 1 shilling.	s.
20 shillings	make 1 pound.	£
21 shillings	make 1 guinea.	

The guinea is equivalent to $5.113 in gold, and the sove-

reign or pound, to $4.87 in gold, or $4.35 in silver. The *nominal par* is $4.44 $\frac{4}{9}$.

France.

10 centimes (c.)	make 1 decime.	d.
10 decimes	make 1 franc.	f.

A franc is equal to 19 $\frac{1}{5}$ cents in gold, or 18 $\frac{3}{4}$ cents in silver. Before the Revolution, the money was divided as follows:

12 deniers	= 1 sol or sou.
20 sous	= 1 livre tournois.
6 livre tournois	= 1 ecu or crown.
4 crowns	= 1 louis d'or.

The livre tournois was equivalent to 18 $\frac{1}{5}$ cents.

Spain.

34 maravedis	make 1 rial.
10 rials of plate	make 1 dollar.
1 rial of plate =	2 rialsvellon.

The Spanish dollar of silver, is of the same value as the American dollar; the dollar of gold is 98 cents. The coin of plate is silver; vellon is the old copper coin of Castile, and is of but half the value.

Portugal.

1000 rees	make 1 milree.
400 rees	make 1 crusado of exchange.

The milree is equal to $1.31 in gold, or $1.12 in silver.

Holland.

100 cents	make 1 florin.

The florin is equal to $0.40 in silver. Accounts are still sometimes kept in Flemish money, the denominations of which are,

6 pennings	make 1 groat.
2 groats	make 1 stiver.
6 stivers	make 1 schilling.
3 $\frac{1}{3}$ schillings	make 1 florin.
6 florins	make 1 pound.

Denmark.

16 skillings	make 1 marc.
6 marcs	make 1 dollar.

The silver dollar, coined before 1813, was equivalent to $1.04. The new, or Rigsbank dollar, is worth but 52 cents.

Hamburgh.

12 pfenings	make 1 sol or schilling lubs.
16 schillings lubs	make 1 marc.
3 marcs	make 1 rix dollar.

The marc current is about 29 cents; the marc banco, about 35 cents, but the currency is exceedingly fluctuating. Lubs denotes the money of Lubec. In exchanges, accounts are frequently kept in Flemish money.

Norway.

120 skillings make 1 species dollar.

The species dollar of silver is equivalent to $1.05. There are no coins of gold.

Sweden.

12 rundstycks	make 1 skilling.
48 skillings	make 1 rix dollar.

The rix dollar banco = 36 cents. There are no coins but copper in circulation.

Russia.

100 copecks	make 1 rouble.

The bank rouble = about 22 cents. The silver rouble = 81 cents. The gold half Imperial, or 5 rouble piece = $3.97.

Prussia.

12 pfenings	make 1 silver groschen
30 silver groschen	make 1 thaler.

The thaler of silver is equivalent to 69$\frac{2}{5}$ cents. Gold coins are bought and sold as merchandise.

Austria.

4 pfenings	make 1 creutzer.
60 creutzers	make 1 florin.
1½ florins	make 1 rix dollar of account.
2 florins	make 1 rix dollar specie.

The silver rix dollar is equal to 96$\frac{2}{5}$ cents. Gold is not legal tender, but circulates only as merchandise.

Naples.

10 grani	make 1 carlino.
10 carlini	make 1 ducato.

The ducat = 79½ cents in silver, or 83 cents in gold.

Sicily.

20 grani	make 1 taro.
10 tari	make 1 ducato.
3 ducati	make 1 oncia.

The ducat is of the same value as that of Naples.

Venice and Genoa.

100 centesimi make 1 lira Italiana.

The lira Italiana is equivalent in value to the franc.

Leghorn.

12 denari	make 1 soldo.
20 soldi	make 1 pezzo.

1 pezzo = 5¾ lire.

The pezzo is equivalent to 90 cents in silver.

Turkey.

3 aspers	make 1 para.
40 paras	make 1 piastre.

The piastre is worth about 8 cents, but its value is exceedingly variable.

East Indies.

4 cowries	make 1 gunda.
20 gundas	make 1 punn.
3 pice	make 1 punn.
4 punns	make 1 anna.
16 annas	make 1 rupee.
16 rupees	make 1 gold mohur.

The gold mohur is equivalent to $8.08;—the Sicca rupee of silver, to 47½ cents;—the current rupee, to about 41 cents. The cowrie is a species of shell. A lac is 100000. A crore is 100 lacs.

Canton.

10 cash	make 1 candarine.
10 candarines	make 1 mace.
10 mace	make 1 tael.

The tael is a weight, about equivalent to 1 oz. 4 dwt. 4 gr., Troy. The tael of silver is usually estimated at $1.48.

By an Act of Congress, the following coins are rendered current in the United States.

The gold coins of Great Britain, Portugal, and Brazil, of not less than 22 carats fine, at $94\frac{8}{10}$ cents per pennyweight.

The gold coins of France, $\frac{9}{10}$ fine, at $93\frac{1}{10}$ cents per pennyweight.

The gold coins of Spain, Mexico, and Columbia, of the fineness of 20 carats $3\frac{7}{16}$ grains, at $89\frac{9}{10}$ cents per pennyweight.

The silver dollars of Mexico, Peru, Chili, and Central America, of not less weight than 415 grains each, and those re-stamped in Brazil, of the like weight, and of not less fineness than 10 oz. 15 dwt. in the Troy pound, of standard silver, at $1.00 each.

The Five-franc piece of France, when of not less fineness than 10 oz. 16 dwt. in the Troy pound, of standard silver,

and weighing not less than 384 grains each, at the rate of 93 cents.

The following table exhibits, nearly, the value of the gold coins included in the Act. But as most of the coins that circulate in the community, are more or less worn, the current value is generally a few cents less.

GREAT BRITAIN.	Guinea, - - -	$5.11
	Sovereign, - -	$4.87
	(Shares in proportion.)	
FRANCE.	Double Louis, coined before 1786,	$9.68
	Double Louis, coined since 1786,	$9.15
	(Single Louis in proportion.)	
	Double Napoleon, or 40 franc piece,	$7.70
	New Louis, and Napoleon, each,	$3.85
SPAIN.	Doubloon of 1772, - -	$16.00
	Pistole, - - -	$3.88
	Coronilla, or gold dollar of 1801,	$0.98
PORTUGAL AND BRAZIL.	Dobraon of 24000 rees, -	$32.70
	Dobra of 12800 rees, -	$17.30
	Johannes, - -	$17.06
	Moiodore, - -	$6.55
	Piece of 16 testoons, or 1600 rees,	$2.12
	Milree of the African Colonies,	$0.78
	Old crusado of 400 rees, -	$0.58
	New crusado of 480 rees. -	$0.64

VIII.

TABLES OF WEIGHTS.

TROY WEIGHT.

24 grains (gr.)	make 1 pennyweight.	dwt.
20 pennyweights	make 1 ounce.	oz.
12 ounces	make 1 pound.	lb.

Apothecaries' Weight.

20 grains (gr.)	make 1 scruple.	℈
3 scruples	make 1 dram.	ʒ
8 drams	make 1 ounce.	℥
12 ounces	make 1 pound.	℔

The pound is the same as the pound Troy. This weight is used only in compounding medicines;—drugs are bought and sold by Avoirdupois Weight.

Diamond Weight.

4 quarters	make 1 grain.
4 grains	make 1 carat.

The diamond grain is $\frac{4}{5}$ of a Troy grain. The gold carat grain = $2\frac{1}{6}$ dwt.

Avoirdupois Weight.

10 grains	make 1 scruple.	sc.
3 scruples	make 1 dram.	dr.
16 drams	make 1 ounce.	oz.
16 ounces	make 1 pound.	lb.
28 pounds	make 1 quarter.	qr.
4 quarters	make 1 hundred-weight.	cwt.
20 hundred-weight	make 1 ton.	T.

The Troy grain = 1.097 grains, Avoirdupois. The pound Avoirdupois, = 7000 grains, Troy.

French Decimal Weight.

10 milligrammes	make 1 centigramme.
10 centigrammes	make 1 decigramme.
10 decigrammes	make 1 Gramme.
10 grammes	make 1 decagramme.
10 decagrammes	make 1 hectogramme.
10 hectogrammes	make 1 kilogramme.
10 kilogrammes	make 1 myriagramme.
10 myriagrammes	make 1 quintal.
10 quintals	make 1 million or bar.

1 gramme = 1.543402 grains, Troy.

The Gramme is a cubical centimetre.

The half kilogramme is called a *livre usuelle*, and the livre is subdivided as follows,

4 gros	make 1 once.
16 onces	make 1 livre usuelle.

IX.

TABLES OF MEASURES.

Long Measure.

3 barleycorns (b. c.)	make 1 inch.	in.
12 inches	make 1 foot.	ft.
3 feet	make 1 yard.	yd.
5.5 yards	make 1 rod or pole.	r.
40 rods	make 1 furlong.	fur.
8 furlongs	make 1 mile.	m.
3 miles	make 1 league.	lea.
69.1 miles, or 60 geographical miles	make 1 degree.	°.

360 degrees make the earth's circumference.

A palm is 3 inches; a hand is 4 inches; a span is 9 inches; a pace is 3 feet; a fathom is 6 feet; a knot is a geographical mile.

The Chinese li	is 632	yards.
The Russian werst	1167	"
The ancient Roman mile	1614	"
The Tuscan mile	1808	"
The Turkish berri	1826	"
The Arabian mile	2148	"
The Irish mile	2240	"
The Polish mile	6075	"
The Persian Parasang	6086	"
The German short mile	6859	"
The Dutch mile	8100	"
The Prussian mile	8237	"
The Danish mile	8244	"
The Dantzic mile	8475	"
The Hungarian mile	9113	"
The Swiss mile	9153	"
The German long mile	10126	"
The Hanover mile	11559	"
The Swedish mile	11700	"

Chain Measure.

7.92 inches	make	1 link.	l.
25 links	make	1 pole.	p.
4 poles	make	1 chain.	ch.
10 chains	make	1 furlong.	fur.
8 furlongs	make	1 mile.	m.

A lot of land measuring 10 chains in length and 1 in breadth, is an acre.

Duodecimal Measure.

12 fourths ('''')	make	1 third.	'''
12 thirds	make	1 second.	''
12 seconds	make	1 prime.	'
12 primes	make	1 unit.	

In the result of any operation, the unit is considered as a linear, a square, or a cubic foot, but in multiplication it must always be remembered that **THE MULTIPLIER IS AN ABSTRACT NUMBER.**

Astronomical, or Circular Measure.

60 seconds ('')	make	1 minute.	'.
60 minutes	make	1 degree.	°.
30 degrees	make	1 sign of the zodiac.	s.
12 signs	make	the orbit of a planet.	

The circumference of *every circle* is divided into 360 degrees, and the degrees are subdivided into minutes and seconds.

Cloth Measure.

2.25 inches	make	1 nail.	na.
4 nails	make	1 quarter.	qr.
4 quarters	make	1 yard.	yd.

39.371	inches make	1 French metre.
47.245	inches make	1 French aune, or ell.
22.98	inches make	1 braccio of Leghorn.
24.8	inches make	1 silk braccio of Venice.
26.46	inches make	1 woollen braccio of Venice.
28	inches make	1 arsheen, or Russian ell.
22.25	inches make	1 Leipsic ell.
27.36	inches make	1 Brabant ell.

French Long Measure.

10 millimetres	make	1 centimetre.
10 centimetres	make	1 decimetre.
10 decimetres	make	1 Metre.
10 metres	make	1 decametre.
10 decametres	make	1 hectometre.
10 hectometres	make	1 kilometre.
10 kilometres	make	1 myriametre.

1 Metre = 39.371 English inches.

The toise usuelle = 2 metres. The pied or foot is $\frac{1}{3}$ of a metre. The aune or ell = $1\frac{1}{5}$ metres, or $47\frac{1}{4}$ English inches. The Metre is one ten-millionth part of the distance from the pole to the equator.

Square Measure.

144	square inches	make 1 square foot.
9	square feet	make 1 square yard.
30.25	square yards	make 1 square rod.
40	square rods	make 1 rood.
4	roods, or	make 1 acre.
10	square chains	make 1 acre.
640	acres	make 1 square mile.

French Square Measure.

The unit is the Are, which is a square decametre, equivalent to 3.9539 square rods. The decimal multiples and sub-multiples, are the same as in the other tables.

Cubic Measure.

1728 cubic inches	make 1 cubic foot.
40 feet of round timber	make 1 ton.
50 feet of hewn timber	make 1 ton.
16 cubic feet	make 1 cord foot.
8 cord feet	make 1 cord.
42 cubic feet	make 1 ton of shipping.

The French unit of cubic measure, is the Stere, which is a cubic metre with the decimal multiples and sub-multiples. The Stere is equivalent to 35.3171 cubic feet.

Common Liquid Measure.

4 gills	make	1 pint.
2 pints	make	1 quart.
4 quarts	make	1 gallon.

The beer gallon contains 282 cubic inches. The wine, or apothecaries' gallon contains 231 cubic inches.

Apothecaries' Measure.

60 minims (♏)	make	1 fluidrachm.	f℥.
8 fluidrachms	make	1 fluidounce.	f℥.
16 fluidounces	make	1 pint.	O.
8 pints	make	1 gallon.	Cong.

A drop of water is *about* equal to 1⅓ minims, or 45 drops, to a fluidrachm. A teacup holds *about* 4 fluidounces; a wineglass, 2 fluidounces; a table spoon, ½ fluidounce; a teaspoon, 1 fluidrachm.

A pint of distilled water weighs 1.04139 lb. avoirdupois.

Dry Measure.

2 pints (pt.)	make	1 quart.	qt.
8 quarts	make	1 peck.	pk.
4 pecks	make	1 bushel.	bu.
36 bushels of coal	make	1 chaldron.	chal.

The dry gallon, or half-peck, contains 268.8 cubic inches.

The Winchester bushel is 18½ inches wide and 8 inches deep.

The French unit of capacity is the Litre, which is a cubic decimetre, equivalent to 1.0517 wine quarts. The boisseau usuel = 12.5 litres. The litron usuel, is the same as the litre, and is equivalent to 1.074 Paris pintes.

Imperial Liquid and Dry Measure.

4 gills	make	1 pint.
2 pints	make	1 quart.
2 quarts	make	1 pottle.
2 pottles	make	1 gallon.
2 gallons	make	1 peck.

4 pecks	make	1 bushel.
4 bushels	make	1 coom.
2 cooms	make	1 quarter.

The gallon "contains 10 lb. avoirdupois weight of distilled water, weighed in air at the temperature of 62° of Fahrenheit's thermometer, the barometer being at 30 inches." This is equivalent to 277.274 cubic inches.

Time Measure.

60 seconds (sec.)	make 1 minute.	min.
60 minutes	make 1 hour.	h.
24 hours	make 1 day.	dy.
7 days	make 1 week.	
30 days	make 1 month.	mo.
365 days	make 1 common year.	y.
366 days	make 1 leap year.	

365 days 5 hours 48 min. 50 sec. make 1 Solar Year.
365 days 6 hours make 1 Julian Year.

The leap years are those that are divisible by 4, as 1844, 1848; except the centurial years, which are leap years when the number of hundreds is divisible by 4, as 1600, 2000, &c. The Julian calendar is still retained in Russia and Greece. This is 12 days later than the New Style, and in leap years, 13 days after the month of February.

X.

MISCELLANEOUS TABLE.

A gallon of train oil	weighs about	7½	lb.
A gallon of vinegar	"	8	lb.
A gallon of molasses	"	11	lb.
A stone of iron, shot, &c.	weighs	14	lb.
A stone of meat	"	5	lb.
A stone of glass	"	8	lb.
A stone of cheese	"	16	lb.
A stone of hemp	"	32	lb.
A fother of lead	"	19½	cwt.

A faggot of steel	weighs	120	lb.
A seam of glass	"	120	lb.
A puncheon of prunes	"	10	cwt.
A firkin of butter	"	56	lb.
A firkin of soap	"	64	lb.
A barrel of soap	"	256	lb.
A barrel of anchovies	"	30	lb.
A barrel of butter	"	224	lb.
A barrel of raisins	"	112	lb.
A barrel of flour	"	196	lb.
A barrel of pork or beef	"	200	lb.
A barrel of shad or salmon	"	200	lb.
A peck of flour or salt	"	14	lb.
A bag of rice	"	168	lb.
A chest of tea	" about	84	lb.
A ton of salt	is	42	bu.
A barrel	holds about	3	bu.

Twelve things	make	1 dozen.
12 dozen	make	1 gross.
12 gross	make	1 great gross.

Twenty things	make	1 score.
5 score	make	1 hundred.
6 score	make	1 long hundred.

24 sheets	make	1 quire.
20 quires	make	1 ream.

A sheet folded into

2 leaves	makes	a folio	fol.
4 leaves	"	a quarto	4to.
8 leaves	"	an octavo	8vo.
12 leaves	"	a duodecimo	12mo.
16 leaves	"	a	16mo.
18 leaves	"	an	18mo.
24 leaves	"	a	24mo.

END OF THE TABLES.

STEREOTYPED BY S. DOUGLAS WYETH,
No. 7 Pear St. Philadelphia.

Sarah M. Sm

1,44139
8
$833112 \div \frac{90}{231} = 3.2458 \times 1$

2250
3000
45000
6250 ... 3600

$75 \times 30 = 2275$
$50 \times 60 = 100$
500×390 1500
1675
62...

1,175
$75 \times 1 = 1 \; 75$
$350 \times 2 = 1 \; 100$
$1500 \times 3 = 4 \; 1500$
43675

625) 1025 (1.2
1250
225
30
12750 ...
1250
250 $\frac{50}{125}$ $\frac{10}{25}$ $\frac{2}{5}$
625